an original Sand Box Geometry Exploratory

ALΣXANDΣR; CEO SAND BOX GEOMETRY

ON THE PARAMETRIC UPGRADE OF EUCLID'S PERPENDICULAR DIVISOR

SPRING

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Using Parametric Geometry, I will construct a Euclidean Perpendicular Divisor to investigate methods of construction that go beyond finding the median of magnitude. I refer to Euclid's Divisor as a Linear Division Assembly and using his precise and exact geometry means to find a linear center; will use this center to build a Curved Space Division Assembly with a Unit Circle and Unit Parabola. We will engage the rules of calculus where the unit parabola is assigned the chores of dependent curvature finding the required slope for placing counting integers of the (y-axis) as iterate linear integer divisors working the (x-axis) measured magnitude of linear (numerator) space into desired parts mandated by the (denominator) counting integers. I will demonstrate utility of identities that populate a Sand Box Geometry Curved Space Division Assembly (CSDA) lending assist to such constructions and show intercept coordinates of a Euclidian Division Hypotenuse with the very familiar primary education Multiplication Table and its multiplication hypotenuse.

An original Sandbox Geometry by ALΣXANDΣR

26 pages. 5000 words

INDEX:

No index yet as this work is under consideration, editing, and advised correction.

BEGGININGS

My first Plane Geometry construction would be dividing a line segment in half. I marveled at the ability of these two tools, a compass and straight edge, to accomplish division of a linear segment (precisely and exactly) going beyond the estimate mechanical observation and mental operation needed with a ruler. I was so impressed I did the routine several times. I am going to write about parametric geometry integer division of linear magnitudes, all will begin with Euclid's perpendicular divisor.

First, let us go back to the fall of 2000 when I decided to take another crack at Calculus 101 at a local community college. Being a person influenced by New Jersey HS grades (9 through 12) in the early 1960's, I was smitten with Big Bang, Quasars, Pulsars, Radio Astronomy, but mostly preoccupied with the space curves Professor Einstein made of Newton's Central Force **F** (how does one curve space?). It was a happening time for young minds as well as popular knowledge. Being introduced to the Cosmos by Dr. Carl Sagan, we were in awe of 20^{th} century astronomy discoveries creating an ever-widening gulf between classical mechanics of Sir Isaac Newton and space curves of the Professor. To understand space curves, I needed to understand how to work the symbols of math. That was my reason for a return to higher education mathematics.

It is during this return to community college I acquired my first "*Mathematica* for Students, ver.4" fall of 2000. For the next several years I examined space curves about a central force **F**. I did so using Computer Algebra graphing utility of parametric geometry; constructing a unit parabola about a unit circle. In so doing, I found that if I let the unit circle be the independent curve and the construct unit parabola be the dependent curve, I could use rules of calculus to check out expected arithmetic behavior of mechanical lines and curves of square space Cartesian Coordinate system by doing constructions with these two elementary plane geometry curves.

Composition of these plane geometry curves constructed as an elementary calculus function can discover and analyze basic linear meter of space curves on a lesser esoteric plane than heavy differential geometry math. I found means to leave behind heavy math and using analytic/parametric geometry pursue understanding *elementary* square space counting mechanics using curves. The unit circle will be my compass and the unit parabola focal radius my straight edge, and armed as the Ancients, let us look at Euclid's perpendicular divisor and find, using his divisor, the inverse square properties of Sir Isaac Newton's Universal Law of Gravity.

Demonstration Construction: parametric description of a Euclidean perpendicular divisor (divide a 3unit line in half):

$$\text{ParametricPlot}[\{\{2\text{Cos}[t],2\text{Sin}[t]\},\{2\text{Cos}[t]+3,2\text{Sin}[t]\},\left\{\frac{3}{2},t\right\}\},\{t,-2\pi,2\pi\},\text{PlotRange}\to\{\left\{-1,\frac{7}{2}\right\},\{-2,2\}\}]$$

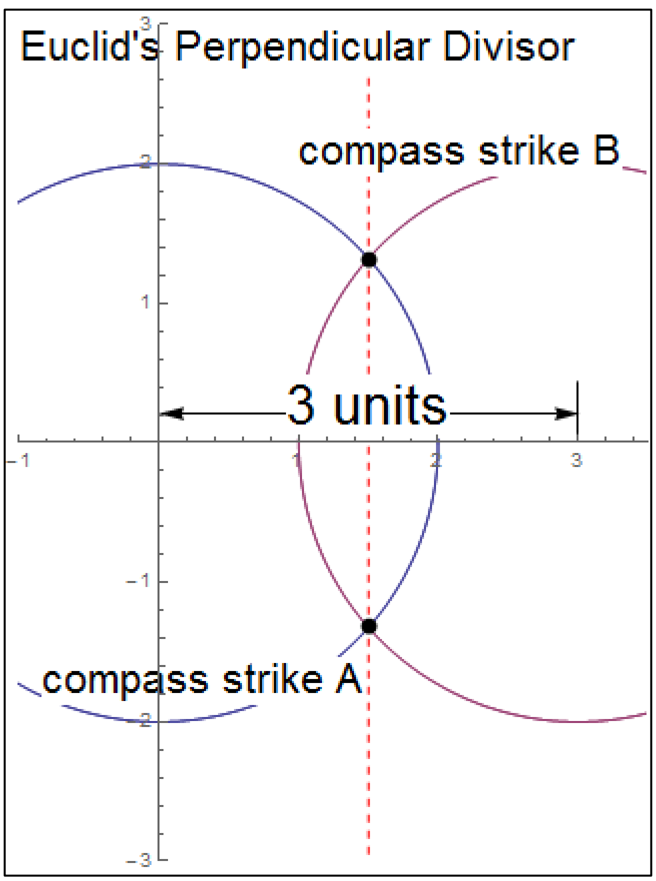

Figure 1: Basic Euclid divisor will divide a given magnitude in half with a perpendicular line construction. Cruzer, Euclidean Upgrade

Constructing the fundamental Euclidean perpendicular divisor on a mag 3-line segment will divide the line segment in half.

This construction involves two compass strikes on unit #3 on the number line. Set the compass radius slightly larger than half the magnitude considered.

Where the compass strikes intersect, we have two points. Connect the points with a straight edge to construct the (normal) line dividing mag 3 in half.

When I construct a Sand Box Geometry **CSDA** on number line point of interest, we find that Euclid's divisor has automatic presence. Radius of the unit circle will be magnitude/2, and when we set builder "p" of the dependent parabola curve equal with "r" of the unit circle, we have an arbitrarily divided segment (two parts) of a given magnitude. We have many reasons to explore magnitude so I have assigned operating distinction to these basic curves beyond independent and dependent 'calculus' moniker. The unit circle will be my discovery curve and the unit parabola will be my definition curve. Half of magnitude containing the origin belongs with the discovery curve circle, the remainder of considered number line magnitude is with the dependent parabola curve. One may ask how we are to conclude the variable "r" to construct our discovery curve? I will have done so with Euclid's divisor. Once I have the midpoint of a line, a parametric **CSDA** is discovered:

$\{Cos[t], Sin[t]\}, \left\{t, \frac{t^2}{-4(p)} \pm (r)\right\}$; where (r) = (p) = $\left(\frac{magnitude}{2}\right)$.

Our knowledge of the world in which we live suffers a great schism. Classical Mechanics belong to Sir Isaac and all his great contemporaries who developed our foundation knowledge. And they are numerous, Galileo, Hook, Euler, Sir Francis Bacon, Kepler, Descartes, the Bernoulli family, Gauss and Riemann, to name just a few people who cultivated the seventeenth century with the seeds of philosophical inquiry concerning the natural surroundings of our being. From the swing of a pendulum to prismatic splitting of sunlight, they laid the foundation math of twentieth century civilization using very sophisticated Plane Geometry Mathematics and thought seeds of Calculus.

Magic of Calculus created the other side of this divide. We find Relativistic Mechanics, Quantum Mechanics, Aerodynamics, Thermodynamics, all from $(F = ma)$, geometry, and calculus.

But the application of Sir Isaac Newton's Universal Law into the micro infinity of Quantum world of matter falls apart causing divide of the human knowledge base into Classical Mechanics and Quantum Relativistic Mechanics.

I intend to return to utility of basic plane geometry to explore mechanical energy curves of two central force fields, (gravity and strong nuclear), by using picture perfect, computer generated parametric geometry as an alternate method to study space curves. Plane Analytic Geometry provides the basic math behind slope and coordinate properties of lines needed to study the scalar multiplication and division (arithmetic) side of space curves. Once the counting of scalar properties of lines and curves, *using* curves is understood, we can begin to meter curved space. And it all begins with Euclid's Perpendicular Divisor. Alexander 2012.

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*INTEGER PARTION OF MAGNITUDE USING A SANDBOX **CSDA***

What better way to study curved space then with curves? I intend to do so using two basic Euclidean Geometry curves. Shared *Central Utility* of center, will produce the required cooperation between two curves needed to demonstrate what we already know about square space math multiplication and its inverse, division.

The players constructing a Curved Space Division Assembly for multiplication and division.

1. There will be only two integer operators: (*integer*) and $(1/integer)$.

2. There will be only two infinities for the two operators to work: micro infinity for inverse integer population and macro infinity for integer population.

3. There can exist one and only one connection between integers found in the macro infinity with the counterpart inverse of integer in the micro infinity due to the principal of curvature and radius of that curvature connectivity.

Principal of linear radii and curvature relativity. For any radius (r) found in the macro infinity there can exist one and only one inverse $\left(\frac{1}{r}\right)$ representation of this radius (as curvature) in the micro infinity. Micro infinity evaluation of radius will be $\left[\left(\frac{1}{r}\right)^{-1} = r\right]$.

My curved space division assembly will use only these two types of (quadrant 1, $(+,+)$) numbers to explore multiplication and the inverse operation division, using curves $(independent\ curvature, dependent\ radius) \xrightarrow{yields} \left(\frac{1}{r},\ r\right)$:

Divide magnitude 3 into 5 parts.

$$\text{ParametricPlot}[\{\{\tfrac{3}{2}\text{Cos}[t], \tfrac{3}{2}\text{Sin}[t]\}, \{t, \frac{t^2}{-4\left(\frac{3}{2}\right)} + \frac{3}{2}\}, \{t, \frac{-5}{3}(t - \frac{3}{5})\}, \{t, \frac{-5}{3}(t - 2(\frac{3}{5}))\}, \{t, \frac{-5}{3}(t - 3(\frac{3}{5}))\}, \{t, \frac{-5}{3}(t - 4(\frac{3}{5}))\}, \{t, \frac{1}{3}(6 - 5t)\}, \{t, \frac{-5}{3}(t - 5(\frac{3}{5}))\}\}, \{t, -2\pi, \frac{9}{2}\}, \text{PlotRange} \to \{\{-1, \frac{7}{2}\}, \{-3, \frac{11}{2}\}\}]$$

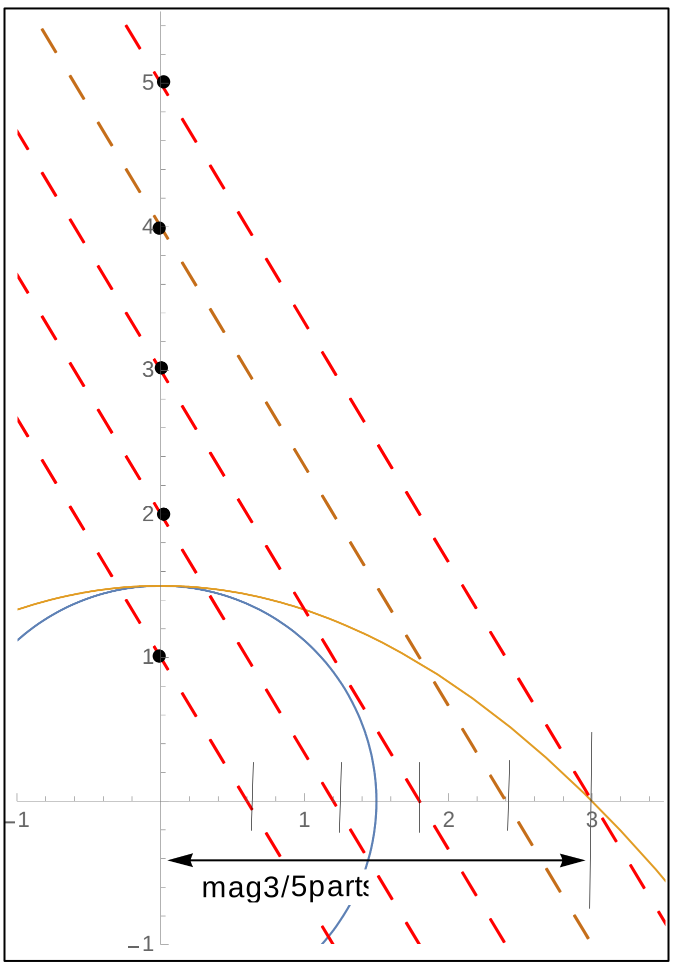


Figure 2: linear magnitude 3 units split into 5 proportional parts.
Cruzer, Euclidean Upgrade

WE use Euclid's perpendicular divisor to split line (3) into two equal parts. Using a compass, construct a circle about the origin using Euclid's median as radius.

$$\left\{\frac{3}{2}\text{Cos}[t], \frac{3}{2}\text{Sin}[t]\right\}$$

Since (p) of the dependent curve is the radius of the independent curve, construct the definition curve out to unit (3) on the number line.

$$\left\{t, \frac{t^2}{-4\left(\frac{3}{2}\right)} + \frac{3}{2}\right\}$$

Let (y) axis integers be counting integers, and (x) axis integers define meter of magnitude. To find slope for iterate divisors, we need the first derivative of the dependent curve$\{t, \frac{t^2}{-4\left(\frac{3}{2}\right)} + \frac{3}{2}\}$.

Ask *Mathematica* for the first derivative term: $\partial_t \left(\frac{t^2}{-4\left(\frac{3}{2}\right)} + \frac{3}{2} \right) \xrightarrow{yields} -\frac{t}{3}$.

All first derivative inquiry of the **CSDA** dependent curve $\left\{\frac{t^2}{-4(p)} \pm (r)\right\}$will be:

$$\left(-\frac{t}{2p}; for\ a\ division\ iterator, let\ (t) be\ number\ of\ parts\right).$$

Iterators are repetitive computer service clones for duplicate lines, all having the same slope intercept with a number line, over and over again. Perfect divisors.

What makes the first derivative term work as an iterator is the position it takes with the dependent curve. The position is called a coefficient part of the dependent operation. It provides slope to the divisor.

A dependent curve divisor term is: $\left(\frac{-t}{3}\left(t - \frac{3}{t}\right)\right)$; where$\left(\frac{-t}{3}\right)$ as coefficient provides slope of divisors where (-t) is number of parts. The term $\left(\frac{3}{t}\right)$ is linear place holder and has magnitude as *numerator* and number of parts as *denominator*. Our first iterator (fig2) becomes:$\left\{t, \frac{-5}{3}\left(t - \left(1 * \frac{3}{5}\right)\right)\right\}$. We have using parametric geometry (t, t); $\left\{t, \frac{-5parts}{3unit}\left(t - \left(1 \times \left(\frac{3unit}{5parts}\right)\right)\right)\right\}$.

Our second divisor becomes: $\left\{t, \frac{-5}{3}\left(t - 2\left(\frac{3}{5}\right)\right)\right\}$, or $\left\{t, \frac{-5parts}{3units}\left(t - 2\left(\frac{3units}{5parts}\right)\right)\right\}$.

Our third divisor becomes: $\left\{t, \frac{-5}{3}\left(t - 3\left(\frac{3}{5}\right)\right)\right\}$ or $\left\{t, \frac{-5parts}{3units}\left(t - 3\left(\frac{3units}{5parts}\right)\right)\right\}$.

Our fourth divisor becomes: $\left\{t, \frac{-5}{3}\left(t - 4\left(\frac{3}{5}\right)\right)\right\}$ or $\left\{t, \frac{-5parts}{3units}\left(t - 4\left(\frac{3units}{5parts}\right)\right)\right\}$.

Our fifth divisor becomes: $\left\{t, \frac{-5}{3}\left(t - 5\left(\frac{3}{5}\right)\right)\right\}$ or $\left\{t, \frac{-5parts}{3units}\left(t - 5\left(\frac{3units}{5parts}\right)\right)\right\}$.

divide a magnitude 5 line into 3 parts:

$$\text{ParametricPlot}[\{\{\frac{5}{2}\text{Cos}[t], \frac{5}{2}\text{Sin}[t]\}, \{t, \frac{t^2}{-4\left(\frac{5}{2}\right)} + \frac{5}{2}\}, \{t, \frac{-3}{5}(t - \frac{5}{3})\},$$
$$\{t, \frac{-3}{5}(t - 2(\frac{5}{3}))\}, \{t, \frac{-3}{5}(t - 3(\frac{5}{3}))\}\}, \{t, -2\pi, 2\pi\},$$
$$\text{PlotRange} \rightarrow \{\{-1,6\}, \{-1,4\}\}]$$

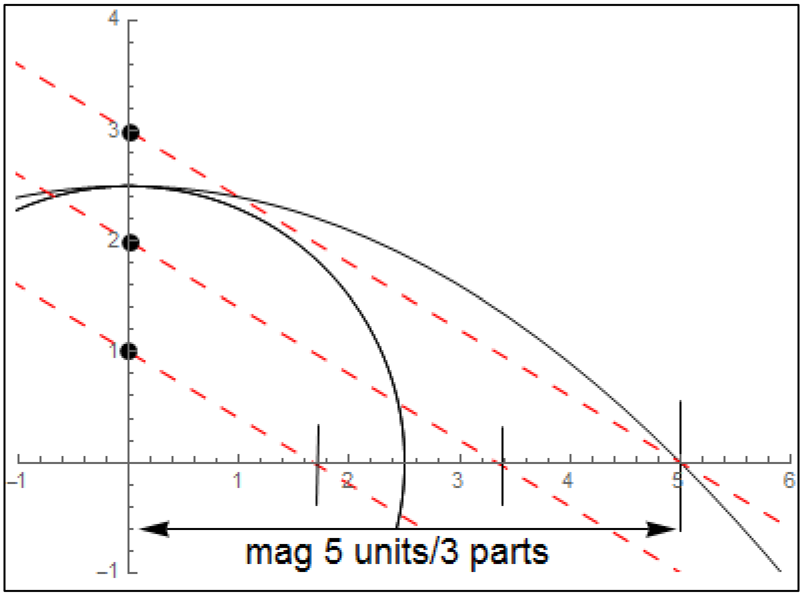


Figure 3: CSDA construction of line 5 units long partition into 3 parts. Cruzer, Euclidean Upgrade

Same procedure as previous, except we will switch linear magnitude and number of partitions.

Using Euclid's divisor, divide linear magnitude (5) in half. Use origin half to construct **CSDA** independent discovery curve. $\left\{\frac{5}{2}\text{Cos}[t], \frac{5}{2}\text{Sin}[t]\right\}$. Next construct the dependent definition framing curve at linear magnitude (5 units), as considered line to be partitioned.

$\left\{t, \frac{t^2}{-4\left(\frac{5}{2}\right)} + \frac{5}{2}\right\}$

The first divisor: $\{t, \frac{-3}{5}(t - \frac{5}{3})\}$; giving us $\left\{t, \frac{-3parts}{5unit}\left(t - \frac{5unit}{3part}\right)\right\}$.

The second divisor: $\{t, \frac{-3}{5}(t - 2\left(\frac{5}{3}\right))\}$; giving us $\left\{t, \frac{-3parts}{5unit}(t - 2\left(\frac{5unit}{3part}\right)\right\}$.

The third divisor: $\{t, \frac{-3}{5}(t - 3\left(\frac{5}{3}\right))\}$; giving us $\left\{t, \frac{-3parts}{5unit}(t - 3\left(\frac{5unit}{3part}\right)\right\}$.

QED: divide a magnitude 5 line into 3 parts

DEMONSTRATION: Construct magnitude 13 divided by 4.

$$\text{ParametricPlot}[\{\{\tfrac{13}{2}\text{Cos}[t],\tfrac{13}{2}\text{Sin}[t]\},\{t,\frac{t^2}{-4\left(\frac{13}{2}\right)}+\frac{13}{2}\},\{t,\frac{-4}{13}(t-\frac{13}{4})\},\{t,\frac{-4}{13}(t-2(\frac{13}{4}))\},$$
$$\{t,\frac{-4}{13}(t-3(\frac{13}{4}))\},\{t,\frac{-4}{13}(t-4(\frac{13}{4}))\},\{t,4\frac{t}{13}\},\{13,t\},\{t,4\}\},\{t,-1,\frac{27}{2}\},$$
$$\text{PlotRange}\text{->}\{\{-1,\frac{27}{2}\},\{-1,8\}\}]$$

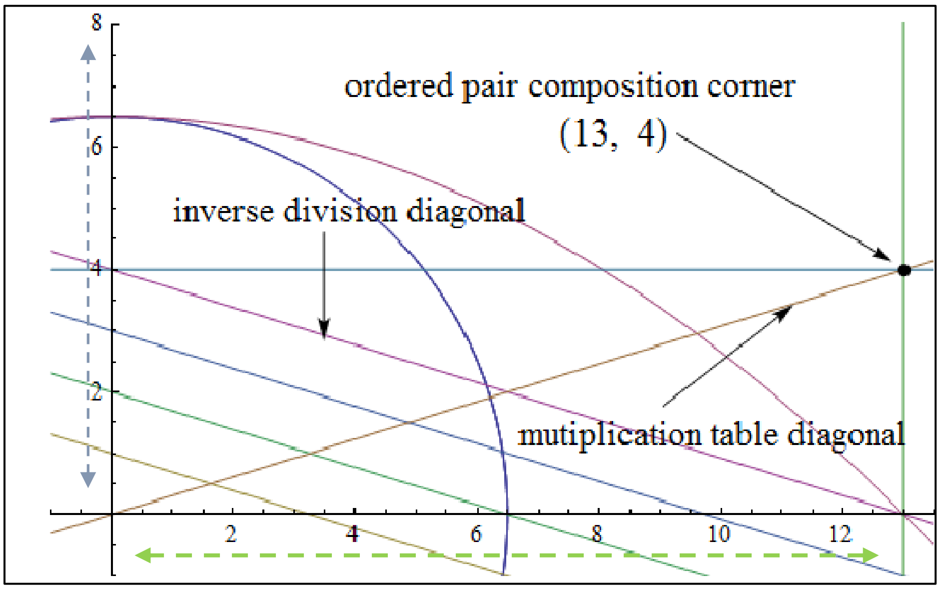


Figure 4: CSDA construction showing multiplication table diagonal and division diagonal.
Cruzer Euclidean Parametric Upgrade

By substituting desired parts of magnitude for numerator (t) in the first derivative term we establish required slope for iterate division diagonals. A division diagonal is plane geometry inverse of the traditional Cartesian multiplication table diagonal connecting system center with square space ordered pair multiplication table composition of square space area $(x * y)$. Division diagonals connect dedicated (y-axis) counting integers as divisors needed to partition considered linear magnitude of number line space.

COMMENTS: We see that a **CSDA** constructing integer division of magnitude is a 1st quadrant square space rectangle displaying a traditional Cartesian multiplication table consisting of 4 (*y axis*) counting units by 13 (*x axis*) meter of linear space. This is a plane geometry description of combined inverse math operators; multiplication (4 x 13) and division (13/4).

A **CSDA** construction for multiplication and division operates using system center as place holder (aka 0, center, or $(0,0)$) in space to build a 1st quad square space perimeter established by (y-axis) counting integer (blue dotted line) and (x-axis) meter of magnitude (green dotted line) figure4. Magnitude, slope, and number of division diagonals, are all assigned to the work horse unit parabola curve. A **CSDA** independent unit circle controls parabola loci growth providing congruent unit meter between lines and curves in both curved and square space. Notice the integer division diagonals connect ordered counting integers with defined partition of magnitude. That side of the rectangle perimeter defining magnitude is greater when magnitude is the greater, and the lesser when magnitude is the lesser, and square when magnitude and integer are equivalent.

In linear mathematics of traditional square space, number 13 divided by four will have a remainder: $\left(\frac{13}{4} = 3\text{remainder}1\right)$. A Euclidean curved space division assembly presents partition of magnitude as a complete sum of the whole by integer parts without remainder.

DEMONSTRATION: *Construct linear magnitude 5 divided by integer 9.*

$$\text{ParametricPlot}[\{\{\frac{5}{2}\text{Cos}[t],\frac{5}{2}\text{Sin}[t]\},\{t,\frac{t^2}{-4\left(\frac{5}{2}\right)}+\frac{5}{2}\},\{t,\frac{-9}{5}(t-(1)(\frac{5}{9})\},$$

$$\left\{t,\frac{-9}{5}\left(t-2\left(\frac{5}{9}\right)\right)\right\},\left\{t,\frac{-9}{5}\left(t-3\left(\frac{5}{9}\right)\right)\right\},\left\{t,\frac{-9}{5}\left(t-4\left(\frac{5}{9}\right)\right)\right\},\left\{t,\frac{-9}{5}\left(t-5\left(\frac{5}{9}\right)\right)\right\},$$

$$\{t,\frac{-9}{5}(t-6(\frac{5}{9}))\},\{t,\frac{-9}{5}(t-7(\frac{5}{9}))\},\{t,\frac{-9}{5}(t-8(\frac{5}{9}))\},\{t,\frac{-9}{5}(t-9(\frac{5}{9}))\}\},$$

$$\{t,-2\pi,\frac{11}{2}\},\text{PlotRange}{-}{>}\ \{\{-1,\frac{11}{2}\},\{-3,10\}\}]$$

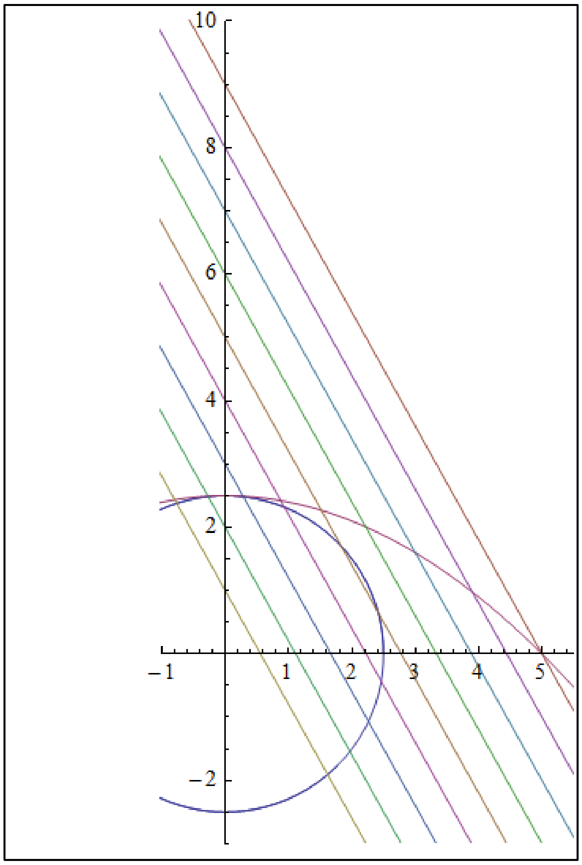


Figure 5:CSDA construction; magnitude (5) into (9) parts.
Cruzer Euclidean Parametric Upgrade

COMMENTS: We see the **CSDA** segments magnitude 5 into 9 integer parts. Notice divisor diagonals collect between system center and final divisor diagonal description to linear magnitude. Whether the greater side of the (4-gon) square perimeter leans to integer divisor or magnitude, a **CSDA** will always sum the divided segments completing the whole at the focal radius 1st quadrant definition of magnitude, where the LATUS RECTUM focal radius will intercept parabola loci curve at slope (m = -1).

END INTEGER DIVISION DEMONSTRATION.

Preliminary visual math perceptions.

- Instead of a one on one integer correspondence to unit partition, we should see each (π/2) collection of integers divided into parts defined by the inverse integer denominator. In other words: magnitude four divided by (1/3) has 12 parts and each (π/2) integer will have 3 of those parts.
- The number of parts will equal: $\{(1/\text{integer})^{-1} * \text{magnitude}\}$.
- Inverse integer division of magnitude will always work between the final division diagonal of a ***perfect square*** and system origin.

CSDA inverse integer division will always operate on a perfect square perimeter. The square root of the number of units within the square perimeter boundary inverted = the *system* CoC; center of curvature anchoring the linear connection joining both infinities with two and only two points, the system CoC (Center of Curvature) with system radius of that curvature. I will show the definition curve focal radius 2(*p*) is actually a curve with curvature $\left(\frac{1}{2p}\right)$! (page 16).

DEFINITION: Curved space connectivity will be a line of specific definition between two end points,framing the dependent curve focal radius with independent curve CoC using curvature terms born of osculating circles. The radius of an osculating circle is used to meter "how much" curvature, or turning, will exist at a point on a curve.

Since *all* iterate inverse integer divisors must sum between a final division diagonal of a perfect square and system origin zero, *all* inversed divisors, of any size will have slope ($m = -1$), slope of all perfect square diagonals.

A sidebar about curvature before inverse integer division.

Every straight line is defined with two points. When a line is used to measure curves, such a line becomes a radius connecting a point on the curve, using conceptual radii of curvature with center of curvature (CoC) as two point linkage.

Since circles always have the same radius, we have shortened radius of curvature for circles, to radius because a circles curvature is constant. To measure curvature not constant on any given curve, we also use two points. 1) The point on the curve, and 2) CoC (center of curvature). The distance between bend and center of curvature has specific meter as radius of curvature. Numerical value for a points curvature is just that; a number only. The usual identity for this bend 'number' is (κ). Defining how 'long' (κ) is? We use radii. We use differential calculus to meter osculating radius of curvature. Something we humans can reason with, a comparative, a unit length, a radius. Something a little more than a dimensionless number.

To find an osculating radius of curvature, we need to construct a tangent (straight line touching the point at its locus) *and* tangent normal. A tangent normal is used to *align* a physical Center of Curvature (CoC) with respect to a locus point and its curvature. Such a (CoC) center found on a constructed tangent normal, will bring three concepts *onto* one straight line known as the tangent normal. (1) The tangent, and (2) intersecting tangent normal @ the point; where *exists* the osculating radius of curvature, and also (3) system (CoC).
A method to evaluate curvature of a point will be found in any first-year calculus text book. Let (κ) be the curvature of a point on our parabola loci;
then: $(\kappa) = \left(\frac{abs|2nd\ derivative|}{((1+1st\ derivative)^2)^{\frac{3}{2}}}\right)$.
In plain language, the above term refers to a fraction. The numerator is an absolute value of the second derivative of a curve. The denominator refers to a sum (1 + the curves first derivative squared) raised to the 3/2 power. If we only work with circles (which I do), this entire operation would mean: to find mathematical description of a circles curvature; simply inverse its radius.

circle	Radius in units	Curvature valuation
A	2	(1/2)
B	(1/2)	2
C	3	(1/3)
D	(1/3)	3

End curvature statement.

A note about dependent CSDA parabola curve evaluation. I have found a parabola offers two evaluations. Difficult differential geometry of curves, and simplistic concentric circle phenomena, such as a stone tossed onto the surface of stilled water. Focal radii trace a moving point on a parabola loci, mimicking circular motion of mechanical energy waves such as a stone tossed into still water. This property is discussed in my paper on Unit Parabola identities.

sandboxgeometry.info duo curve analytics of a parabola.

DEMONSTRATION: The following construction is an inverse integer division plane: 3/ (1/3) = 9 parts:

$$\text{ParametricPlot}[\{\{\tfrac{3}{2}\text{Cos}[t],\tfrac{3}{2}\text{Sin}[t]\},\{t,\frac{t^2}{-4\left(\frac{3}{2}\right)}+\frac{3}{2}\},\{t,-1(t-\tfrac{1}{3})\},\{t,-1(t-\left(\tfrac{2}{3}\right)\},\{t,-1(t-\left(\tfrac{3}{3}\right))\},$$
$$\left\{t,-1\left(t-\left(\tfrac{4}{3}\right)\right)\right\},\left\{t,-1\left(t-\left(\tfrac{5}{3}\right)\right)\right\},\left\{t,-1\left(t-\left(\tfrac{6}{3}\right)\right)\right\},\left\{t,-1\left(t-\left(\tfrac{7}{3}\right)\right)\right\},\left\{t,-1\left(t-\left(\tfrac{8}{3}\right)\right)\right\},$$
$$\{t,-1(t-(\tfrac{9}{3}))\}\},\{t,-2\pi,4\},\text{PlotRange}\text{->}\{\{-1,4\},\{-1,4\}\}]$$

Notice committed integer population on the (π/2) spin radii are divided into inverse integer parts as expected, (integers 1, 2, and 3) into 3 parts each. All inverse integer division hypotenuses pose the same slope (m = – 1) causing the **CSDA** assembly to manufacture the required parts of inverse division between a perfect square diagonal and system origin on the Cartesian plane. We see by inspection both **CSDA** curves work together to define considered magnitude using two infinities. Independent unit circle radius (3/2) constructs the boundary for the micro infinity $\left(\frac{1}{r}\right)$ where we find initial curvature value (CoC: (1/3, 0)) composing the linear connection of curved space and square space (radius 3 unit system) with the dependent unit parabola radius latus rectum connect at (3, 0) in open macro infinity.

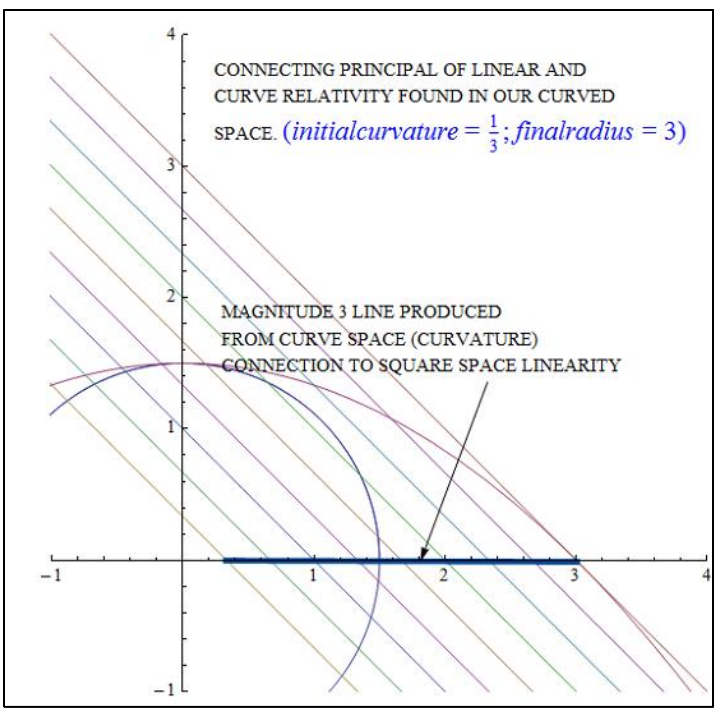


Figure 6:CSDA construction of an inverse integer partition.(3/(1/3)). Cruzer,Euclid Upgrade

Means to create inversed Iterators for division

All coefficient terms for iterated inversed divisors are $(m = -1)$ slope. The placeholder for each divisor terms are:

$$\left((-1slope)\times\left(t-first\ divisor\ \times\left(\tfrac{1}{n}\right)\right)\right),$$

where (n) is inverse integer and number of divisors will be:

$\left(\left(\frac{1}{n}\right)^{-1} \times (magnitude)\right)$.

Each divider of an inverse integer construction has constant slope $(m = -1)$ to establish slope for division Iterators. Each divider; divisor 1, divisor 2, divisor 3 ... divisor $\left(\left(\frac{1}{n}\right)^{-1} \times (magnitude)\right)$ establish place holder or position of each divisor. $\left(\frac{(divisor\#)\times\left(\frac{1}{n}\right)}{magnitude}\right)$ **CSDA** inverse divisor have specific relation with the denominator as magnitude. For this example, we have thirds giving us 9 parts $\left(\left(\frac{1}{3}\right)^{-1} \times (3) = 9\right)$. Note that the numerator of each place holder, beginning with (1/3) increase the *numerator* by 1 going from 1 to 9 to define each divisor. When the inverse integer *numerator* reaches 9, the *denominator* 3 will realize the final divisor, as a fraction, will become partition magnitude.

1. 1st divisor: $\left\{t, -1\left(t - \frac{1}{3}\right)\right\} \xrightarrow{yields} 1st\ third$
2. 2nd divisor: $\{t, -1\left(t - \left(\frac{2}{3}\right)\right)\} \xrightarrow{yields} 2nd\ third$
3. 3rd divisor: $\left\{t, -1\left(t - \left(\frac{3}{3}\right)\right)\right\} \xrightarrow{yields} 3rd\ third$
4. 9th divisor: $\left\{t, -1\left(t - \left(\frac{9}{3}\right)\right)\right\} \xrightarrow{yields} 9th\ third:$
   END iterators; all (fig.6) divisors accounted for.

Dual curve analytics of a parabola

I mentioned concentric wave form as alternate curve evaluation for a parabola focal radius. The next two constructions shows both differential geometry meter of curvature for loci point on a parabola and **CSDA** concentric circle wave form meter of a central force.

The focal radii (u1) and (u2), point out vertices of two curve analytic tangents and their normal. Since a radius of curvature lies on a tangent normal with the CoC of loci curvature, an osculating curve can be constructed at the CoC. However, we don't know where to position an osculating curve CoC with respect to a point's vertex curvature.

I can use math to determine a parabola loci curvature, know the radius of curvature and construct the osculating curve at the point and *see* the radius of curvature as an intercept on the curves tangent normal.

The problem with differential geometry meter of radius of curvature on the tangent normal, is that these radii of curvature don't serve well wave form emanating from a central force origin (**F**).

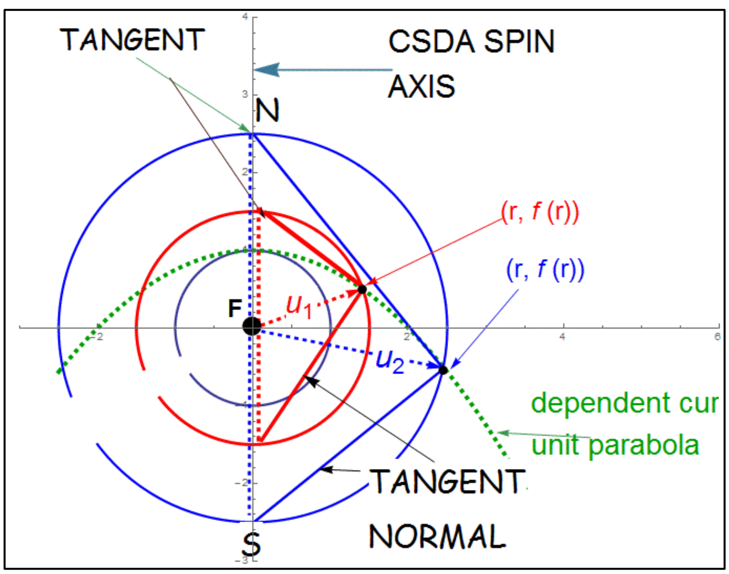
$$\text{ParametricPlot}[\{\{1\text{Cos}[t],1\text{Sin}[t]\},\{\frac{5\text{Cos}[t]}{4},\frac{5\text{Sin}[t]}{4}\},\{t,-\frac{t^2}{4}+1\},\{t,\frac{t^2}{+4}-1\},$$

$$\{1,t\},\left\{t,\frac{1}{4}(5-2t)\right\},\left\{t,\frac{1}{4}(-5+8t)\right\},\left\{\frac{1}{4}(5\sqrt{5})\text{Cos}[t]+1,\frac{1}{4}(5\sqrt{5})\text{Sin}[t]+\frac{3}{4}\right\},$$

$$\left\{\frac{-1}{4},t\right\},\left\{t,\frac{-7}{4}\right\},\{\frac{1}{4}(5\sqrt{5})\text{Cos}[t]-\frac{1}{4},\frac{1}{4}(5\sqrt{5})\text{Sin}[t]-\frac{7}{4}\}\},\{t,-2\pi,2\pi\},$$

$$\text{PlotRange}\to\{\{-5,4\},\{-5,4\}\}]$$

Focal radii (u1) and (u2), on the other hand, serve as new central relative radii, providing new circles of curvature as visible by connected colors, red circle for (u1) and blue circle for (u2). As the parabola focal radius sweeps the locus of the curve, new curve meter events occur as position $(r, f(r))$ of the focal radius vector head are realized. Differential geometry CoC must change with each osculating circle constructed on parabola loci, why not just use 2500-year-old focal radii description of changing wave form energy emanating from center **F**.



I present a construction showing difficulty of finding curves CoC using a parabola. The point considered on the parabola curve is $(1, 3/4)$.

Using higher math, I find the curvature at $(1, 3/4)$ of a unit parabola is $\left(\frac{4}{5\sqrt{5}}\right)$; inversed will give a radius of curvature for an osculating curve $\left(\frac{5\sqrt{5}}{4}\right)$. The focal radius of the construction points to vertices of tangent and tangent normal at $(1, 3/4)$, and also gives a new

Figure 7: concentric wave form emanating from (**F**)

(curve) circle: $\left\{\frac{5\mathrm{Cos}[t]}{4}, \frac{5\mathrm{Sin}[t]}{4}\right\}$. Using higher math I have determined CoC for $(1, 3/4)$ and constructed the osculating curve for vertex point $(1, 3/4)$; at the CoC: $\left\{\frac{1}{4}(5\sqrt{5})\mathrm{Cos}[t] - \frac{1}{4}, \frac{1}{4}(5\sqrt{5})\mathrm{Sin}[t] - \frac{7}{4}\right\}$. Since I know *both* loci point of curvature connectivity (CoC *and* $(1, 3/4)$); for point $(1, 3/4)$ there is no reason why I can't move the osculating curve from CoC $\left(-\frac{1}{4}, -\frac{7}{4}\right)$ to $(1, 3/4)$, $\left\{\frac{1}{4}(5\sqrt{5})\mathrm{Cos}[t] + 1, \frac{1}{4}(5\sqrt{5})\mathrm{Sin}[t] + \frac{3}{4}\right\}$ and see radius of curvature intercept with CoC on tangent normal.

QED Alexander.

I have constructed two osculating curves for $(1, 3/4)$.

The black hash mark curve is $(1, 3/4)$ osculating curve positioned at CoC of $(1, 3/4)$.

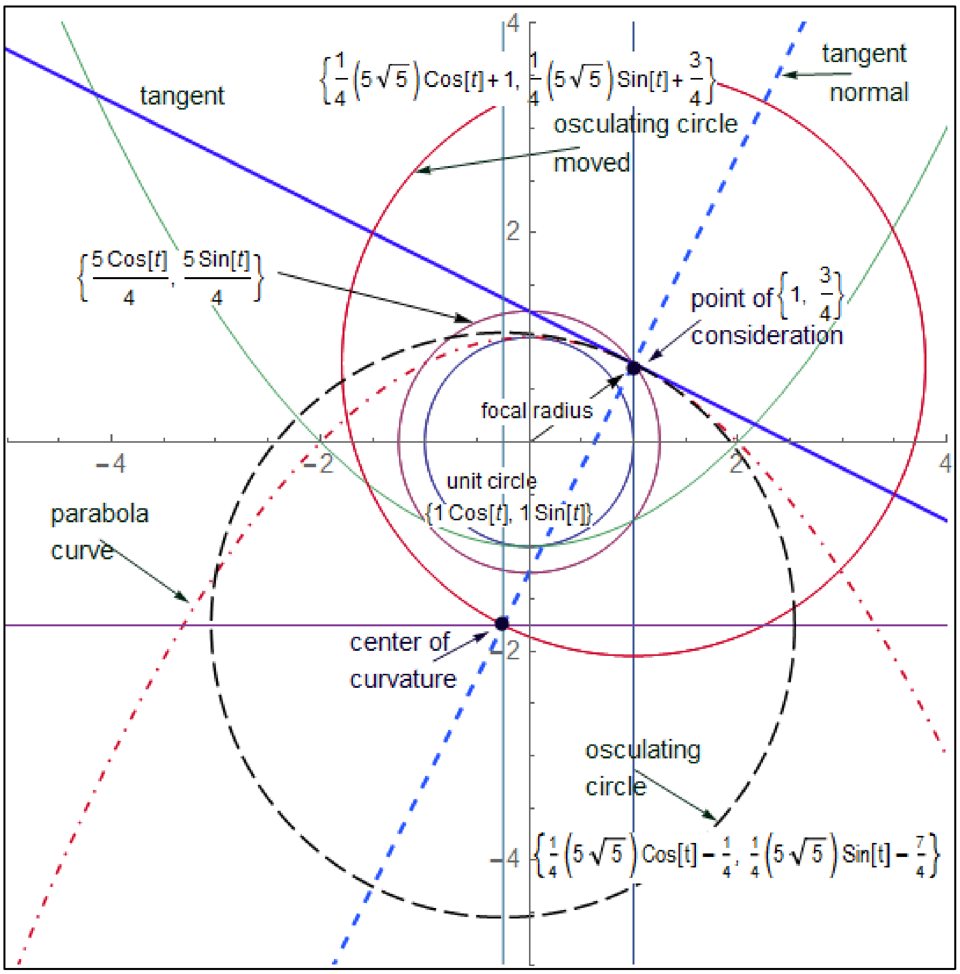


Figure 8: demonstration of dual curve analytic geometry of a parabola.

The solid red osculating curve is radius of curvature for $(1, 3/4)$ placed at $(1, 3/4)$. Differential geometry is perfect to help higher math meter curvature evaluation of loci points, but fail in methods to meter central force motivated (**F**) curves. In the previous example, the inverse integer division denominator number is equal with unit magnitude number producing a perfect square where system radius of curvature (3), has connection with expected CoC

(1/3 as inversed r =3) found in the micro infinity. This connection is a required tool for plane geometry discovery of meter concerning lines and curves found in central force (**F**) constructions using space curves.

The next example will display a division assembly with unlike numbers, integer division denominator number ≠ magnitude number. The **CSDA** divisors structuring the curved space Connection Principal seems to fail. Here is a parametric description for a **CSDA** for magnitude (8) divided by (1/2).

DEMONSTRATION: Linear magnitude (8) divided by inverse of integer (2) will have 16 division diagonals. In this demonstration only the first six and the last three division diagonals are displayed.

$$\text{ParametricPlot}[\{\{4\text{Cos}[t],4\text{Sin}[t]\},\{t,\frac{t^2}{-4\left(\frac{8}{2}\right)}+4\},\{t,-1(t-\frac{1}{2})\},\{t,-1(t-\frac{2}{2})\},$$
$$\left\{t,-1\left(t-\frac{3}{2}\right)\right\},\left\{t,-1\left(t-\frac{4}{2}\right)\right\},\left\{t,-1\left(t-\frac{5}{2}\right)\right\},\left\{t,-1\left(t-\frac{6}{2}\right)\right\},\left\{t,-1\left(t-\frac{14}{2}\right)\right\},$$
$$\{t,-1(t-\frac{15}{2})\},\{t,-1(t-\frac{16}{2})\}\},\{t,-\frac{1}{2},\frac{17}{2}\},\text{PlotRange}\text{->}\{\{-1,9\},\{-3,\frac{17}{2}\}\}]$$

Divisors one through six and the final three of 16 divisors are constructed. Every divisor will have slope (m = –1). Notice each π/2 integer is divided in half, splitting magnitude (8) into (16) equal parts. This example will sum the parts (16 total) constructing the whole (8) at the curved space unity ratio where all three events, tangent slope of definition curve, linear magnitude, and integer parts summing linear magnitude, meet at slope $(m = -1)$.

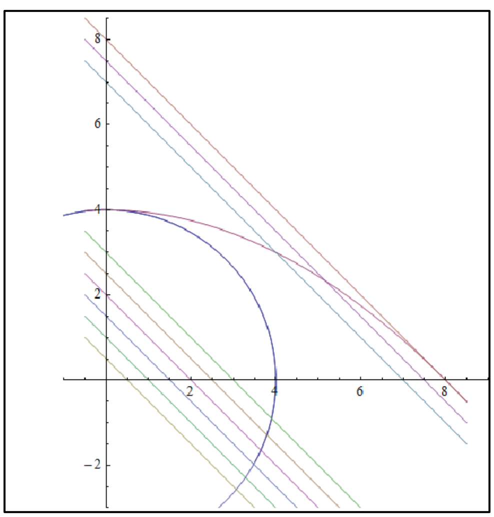


Figure 9: CSDA construction of linear magnitude divided by inversed integer 2 or(1/2). Cruzer, Euclid parametric upgrade

All inverse integers working a **CSDA** sum a perfect square, but our dual infinity operating terms, ½ and 8, leaves Curved Space Connection Principal out of "focus", we expect $\left(CoC = \frac{1}{8}\right)$ because (**CSDA** r = 8).The final **CSDA** division diagonal for magnitude (8) is at slope (m = –1), but the *first* initial inverse divisor apparently claiming curve connectivity is (1/2) and not (1/8) as (CoC) for CSDA radius (8), a seeming incongruity. How do we find correction within the square table of math operators division and multiplication to satisfy the Connecting Principal found in our curved space?

A Postulate for Initial(CoC)Curvature and Final Radius Connecting Principal of a CSDA: Center of Curvature of every **CSDA** system analysis will always be: [(Independent Curve Curvature) times the Euclidean constant (1/2) used to split space into the twin infinities of our being].

For our example of 8 unit magnitude into halves: **CSDA** COC for mag8units/2:

$$\left(independent\ radius = 4, \left(k = \frac{1}{4}\right); and\ system\ center\ of\ curvature = \left(\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}\right)\right)$$

If the independent **CSDA** circle radius is (magnitude/2) then **CSDA** circle curvature is (2/magnitude); and (1/2) ×(2/magnitude) = $\left(\frac{1}{2} \times \frac{2}{magnitude} = \frac{1}{magnitude}\right)$ as **CSDA** system (CoC), and $\left(\left(\frac{1}{magnitude}\right)^{-1} = magnitude\right)$

The previous construction demonstrates the fact we have two radii of meter to do curve analytics. One for square space and one for curved space. In square space analytics, Cartesian center $(0, 0)$ is fixed with the constant curve loci of Archimedes circle. However, once the curve is made not constant, differential geometry provides a new radius, not one with an easily fixed and discernible center, but a radius based on CoC (center of curvature) and RoC (radius of curvature) found with the math of osculating curves. The plane analytic parametric geometry of my **CSDA** dependent curve can handle both.
QED END Curved Space Partition of Magnitude

Conclusion: We have introduced two radii of measure in our mathematical tool box. One is the radius of Archimedes. The other has been born of heavy math of Gauss and Riemann vision of Differential Geometry. The seeds of Curvature exploration using Differential Geometry were cultivated by many prior to their (Gauss and Riemann) advancement. I know this because Sir Isaac Newton[1] certainly knew of curvature because he coated a most difficult concept (curvature) as inverse square law $\left(\frac{1}{r^2}\right)$; which is the same as $(curvature)^2$.

Calculus, or as he called his work fluxions, was enough removed (somethin' new) from mainstream knowledge (bringing expected criticism and debate) that he coated $(curvature)^2$ as inverse square law, a developing explored concept in his time.

The two radius of meter:

- Archimedes: from origin center to circle (our square space radius).
- Our curved space radius: From osculating center of curvature (CoC), as one end point *part* of a radius of curvature. The other end point part is found on the considered loci of a curve as final concluding end point, not as a radius, but an osculating circle radius of curvature. Osculating radii have specific meter, derived from osculating curvature value(κ), a number. To realize a linear value as a physical measure we can 'see', we invert curvature to acquire radius of curvature at a point.

ALΣXANDΣR; CEO SAND BOX GEOMETRY LLC

An aside: Using higher math, I find the curvature at $(1, 3/4)$ of a unit parabola is $\left(\frac{4}{5\sqrt{5}}\right)$; and the radius of curvature for $(1, 3/4)\ is\ \left(\frac{5\sqrt{5}}{4}\right)$. Now, the focalradius at $(1, 3/4)$ is $(2 - f(r))$ $\xrightarrow{yields} \left(2 - \frac{3}{4}\right) \xrightarrow{yields} \left(\frac{5}{4}\right)$. A CSDA focal radius can be used to find differential geometry curvature evaluation of parabola loci (where unit 2 is parabola vertex RoC; 2p).

$$(2)\left(\frac{5}{4}\right)\left(\sqrt{\frac{5}{4}}\right) = \frac{5\sqrt{5}}{4}; \boldsymbol{CSDA}\ system\ RoC$$

[1]Alfred Gray, Modern Differential Geometry of Curves and Surfaces, with Mathematica, footnote page 14

Curvature and radius of curvature

From my Hartford Connecticut MAA MathFest 2013 paticipation.

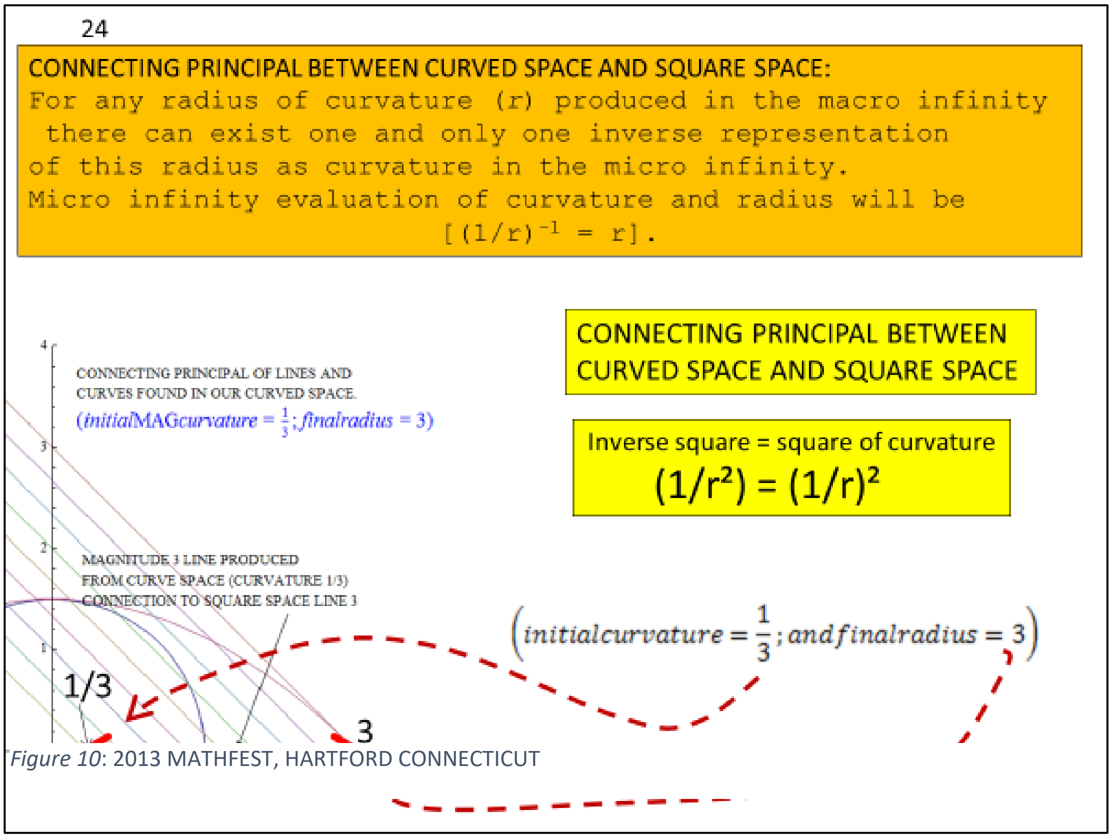


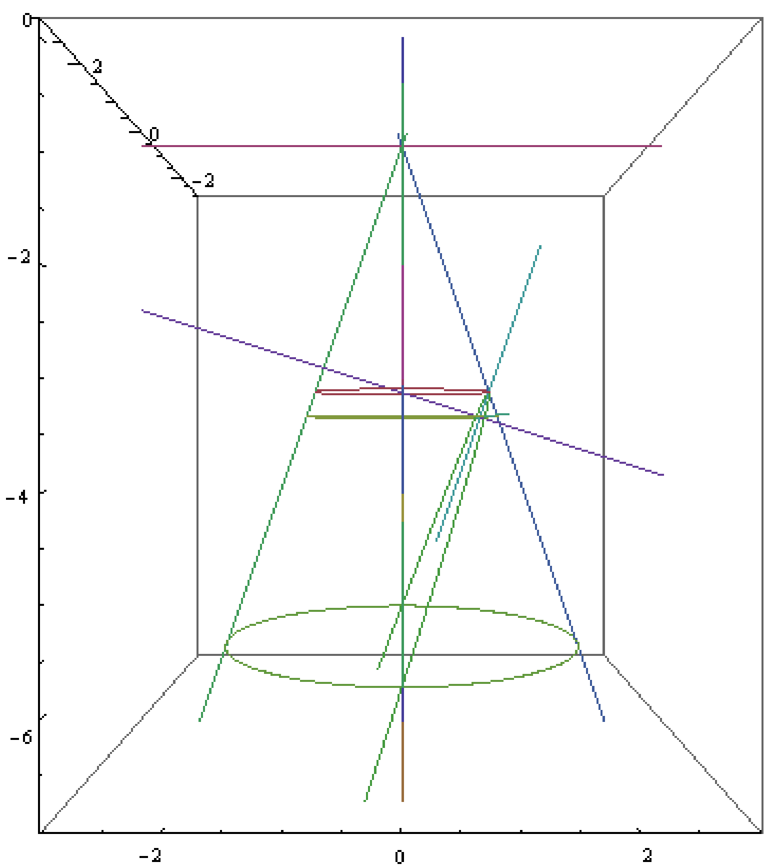
Figure 10: 2013 MATHFEST, HARTFORD CONNECTICUT

WHY FUSS WITH A CONNECTING PRINCIPAL? Because Sir Isaac Newton has given us inverse square space to explain field properties we live with and inverse square law is essentially the square of curvature.

Sand Box Geometry LLC, a company dedicated to utility of Ancient Greek Geometry in pursuing exploration and discovery of Central Force Field Curves.

Using computer parametric geometry code to construct the focus of an Apollonian parabola section within a right cone.

“It is remarkable that the directrix does not appear at all in Apollonius great treatise on conics. The focal properties of the central conics are given by Apollonius, but the foci are obtained in a different way, without any reference to the directrix; the focus of the parabola does not appear at all... Sir Thomas Heath: “A HISTORY OF GREEK MATHEMATICS” page 119, book II.



Utility of a Unit Circle and Construct Function Unit Parabola may not be used without written permission of my publishing company Sand Box Geometry LLC Alexander; CEO and copyright owner.alexander@sandboxgeometry.com

The computer is my sandbox, the unit circle my compass, and the focal radius of the unit parabola my straight edge.

ALΣXANDΣR; CEO SAND BOX GEOMETRY LLC

CAGE FREE THINKIN' FROM THE SAND BOX

The square space hypotenuse of Pythagoras is the secant connecting (π/2) spin radius (0, 1) with accretion point (2, 0). I will use the curved space hypotenuse, also connecting spin radius (π/2) with accretion point (2, 0), to analyze g-field mechanical energy curves.

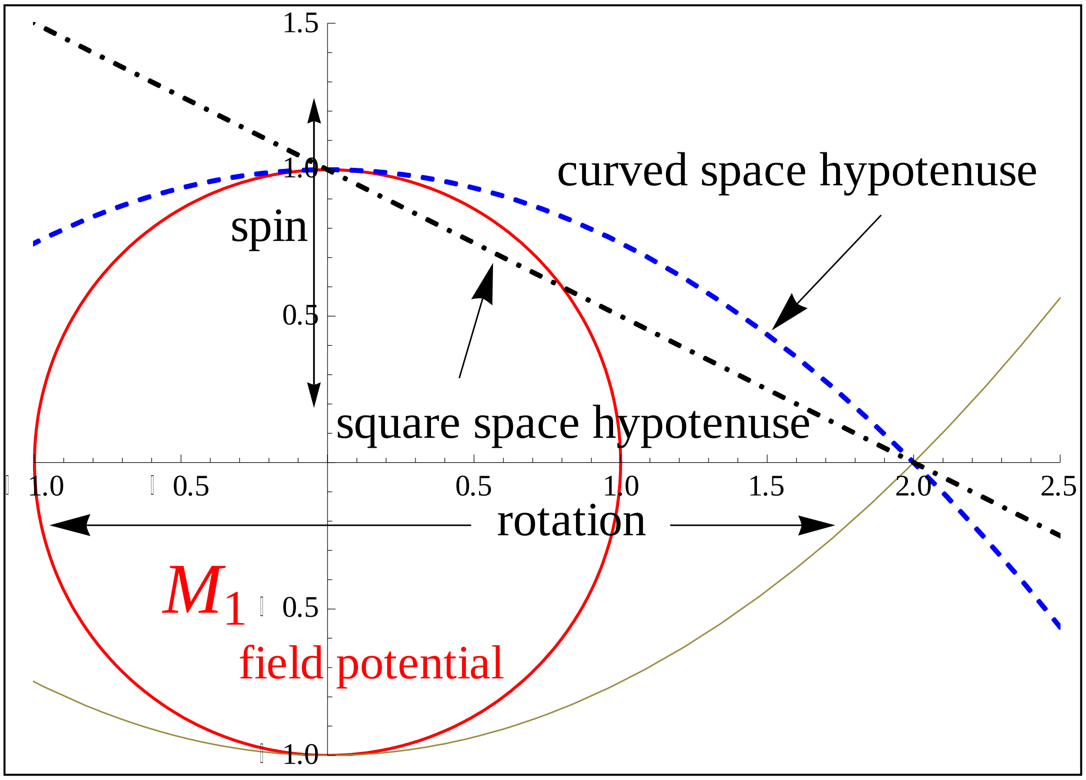


*Figure 11: **CSDA** demonstration of a curved space hypotenuse and a square space hypotenuse together.*

We have two curved space hypotenuses because the gravity field is a symmetrical central force, and will have an energy curve at the **N** pole and one at the **S** pole of spin; just as a bar magnet. When exploring changing acceleration energy curves of M_2 orbits, we will use the N curve as our planet group approaches high energy perihelion on the north time/energy curve.

ALΣXANDΣR; CEO SAND BOX GEOMETRY