Sand Box Explanatory

Source and Construction of Independent Curve Unit Circle and Dependent Curve Unit Parabola Within an Apollonian Conic Section. Precursor and origin of **CSDA** mechanical construction and analytics of Central Force Mechanical Energy Curves.

ALΣΧΑΝDΣR; CEO SAND BOX GEOMETRY LLC

# Exploring an Apollonian Conic CSDA

June 27
2019

Part 2 of Unit Parabola explanatory. This paper examines source primitive of the dependent CSDA curve Unit Parabola.

Computer based math construction protocols for Apollonian Conic Section Parabola CSDA

Thursday, June 27, 2019

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Using *Mathematica* and Cartesian Coordinates to construct an Apollonian parabola section onto a cone surface with parametric based computer geometry.

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**STEP 1.** Construct a cone central axis with generator slope (±2). Set cone nappe below horizontal axis to aquire traditional view.

$$ParametricPlot[\{\{t, 2t\}, \{t, -2t\}\}, \{t, -\pi, \pi\},$$
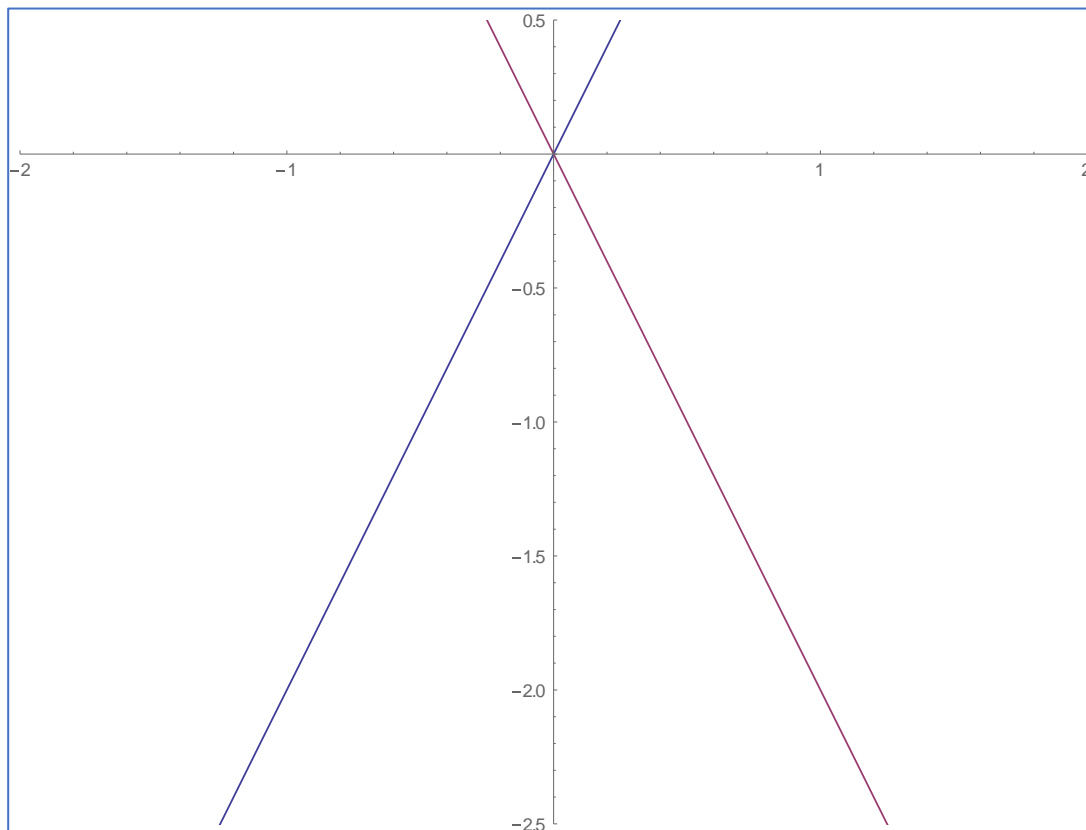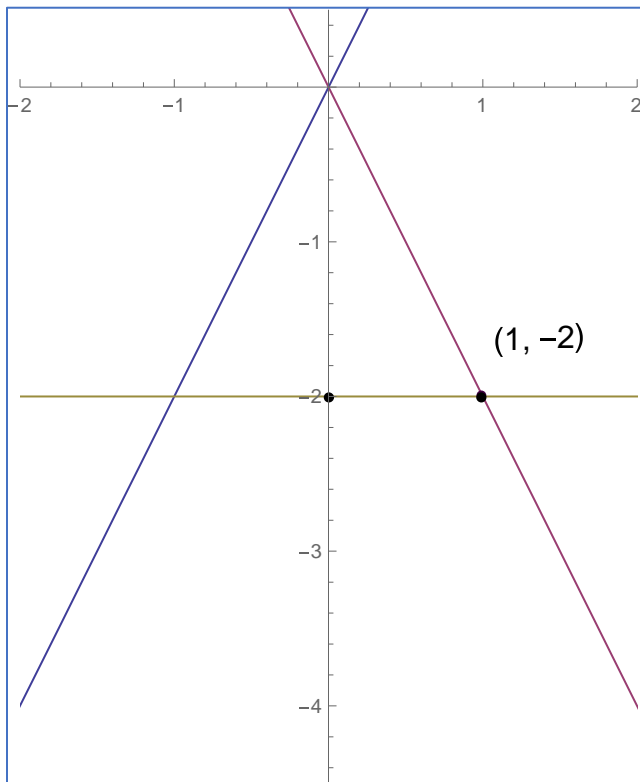
$$PlotRange-> \{\{-2,2\}, \{-5/2,1/2\}\}]$$



*Figure 1:* lower nappe of slope (*m* = 2) cone.

# **STEP 2.** Construct the unit circle diameter.

SBG theorem: When cone altitude (A) is numerically congruent with slope (m) we have a unit circle diameter. ∴ unit circle parametric diameter will be {t, -2}.

$$ParametricPlot[\{\{t, 2t\}, \{t, -2t\}, \{t, -2\}\}, \{t, -\pi, \pi\},$$

$$PlotRange-> \{\{-2,2\}, \{-9/2,1/2\}\}]$$



(1, –2)

Go down 2 cone units to find a unit circle diameter constructed within a slope ($m = 2$) cone.

*Figure 2:* finding a unit circle conic diameter.

**STEP 3.** Mapping a parabola principal axis onto a cone with section vertex on a unit circle diameter: position the parabola section vertex at Cartesian Coordinate (point (1, -2)).

$$Solve[y - (-2) == 2(x - 1), y] = \{y \rightarrow 2(-2 + x)\}$$

$$convert\ algebraic\ solution\ (2(-2 + x))\ to\ parametric\ description \xrightarrow{yields} \{t, -4 + 2t\}$$

$$ParametricPlot[\{\{t, 2t\}, \{t, -2t\}, \{t, -2\}, \{t, -4 + 2t\}\},$$
$$\{t, -\pi, \pi\}, PlotRange-> \{\{-2,2\}, \{-9/2,1/2\}\}]$$
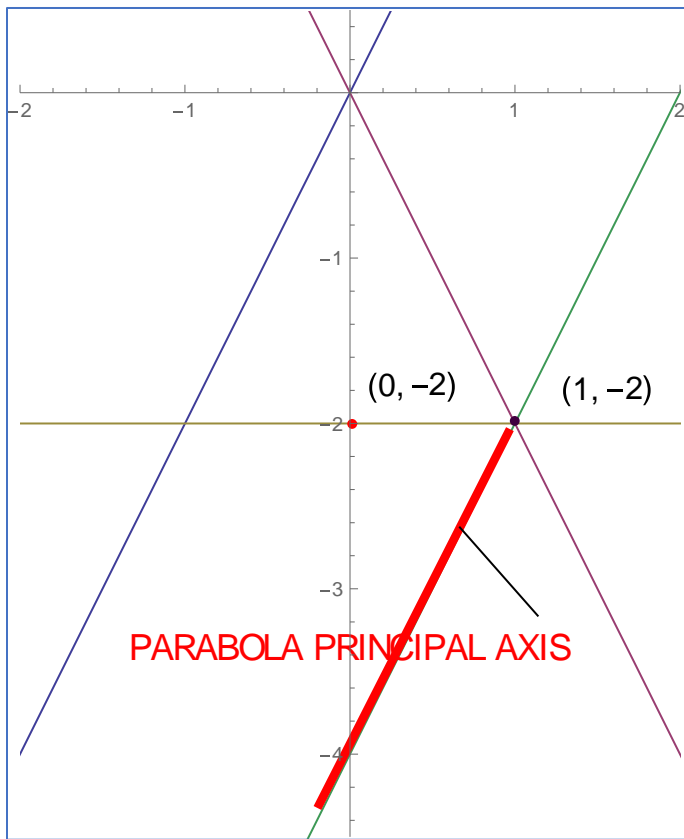


*Figure 3*: finding a parabola section principal axis.

A cone parabola section will have its principal axis parallel with one and only one generator. Let the section vertex be generator $\left(\frac{\pi}{2}\right)$. Construct relative with generator $\left(\frac{3\pi}{2}\right)$; parametric $\{t, 2t\}$. This construction will set the parabola vertex at the positive endpoint profile view unit circle diameter.

Using the section vertex $(1, -2)$ as point of reference and opposing generator $\{t, 2t\}$ as slope, a linear point slope description of the apollonian section parabola principal axis can be found. convert the algebraic solution to parametric description and post to the construction. The sections principal axis is in red.

**STEP 4.** Construct the parabola focal axis.

**SBG** Theorem: every parabola focal axis is normal to the parallel generator $\{t, 2t\}$ and will pass through unit circle diameter center. (unit circle center is (0, -2)).

$$Solve[y - (-2) == -(2) - 1(x - 0), y] \xrightarrow{yields} \left\{\left\{y \rightarrow \frac{1}{2}(-4 - x)\right\}\right\}$$

Convert algebra $\left(\frac{1}{2}(-4 - x)\right)$ to parametric description; $\{t, 1/2 \ (-4 - t)\}$

$$ParametricPlot[\{\{t, 2t\}, \{t, -2t\}, \{t, -2\}, \{t, -4 + 2t\},$$

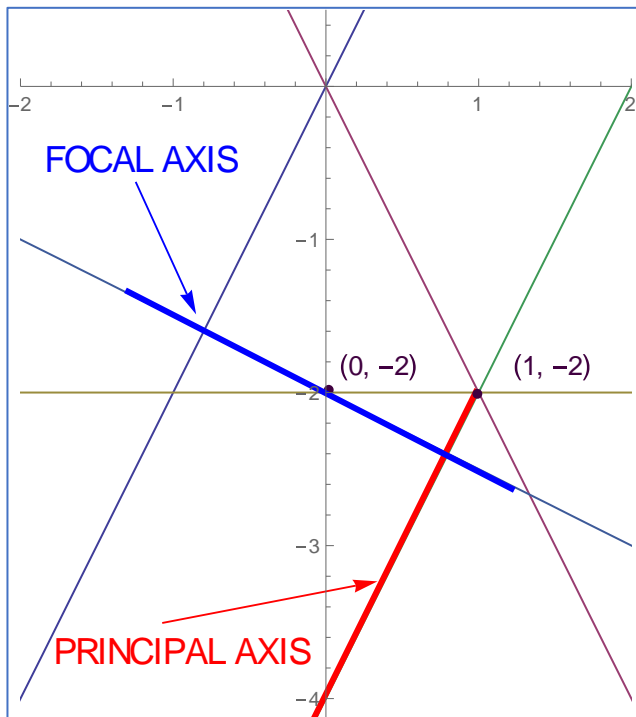$$\{t, 1/2 \ (-4 - t)\}\}, \{t, -\pi, \pi\}, PlotRange-> \{\{-2,2\}, \{-9/2, 1/2\}\}]$$



*Figure 4:* Constructing the focal axis.

Using the section vertex $(1, -2)$ as point of reference and opposing generator $\{t, 2t\}$ as slope, a linear point slope description of the apollonian section parabola principal axis can be found. convert the algebraic solution to parametric description and post to the construction. the sections principal axis is in red, focal axis is blue.

The ancients described the earliest parabola as the section with its principal axis normal with a right cone generator legs having slope (m = ±1). This concept is correct for every parabola section of any cone slope when the focal axis is made normal with the opposing parallel generator following the section principal axis. To find the focal axis parameter use the center of the diameter holding the vertex as point of reference for point slope description. Every parabola focus can be found at the intersection of the principal axis and focal axis. Next we find the neighborhood of (*p*) to determine size of the LATUS RECTUM.

**STEP 5.** Next we determine how far the vertex is from the parabola section focus which is the initial focal radius by constructing the neighborhood of ($p$).
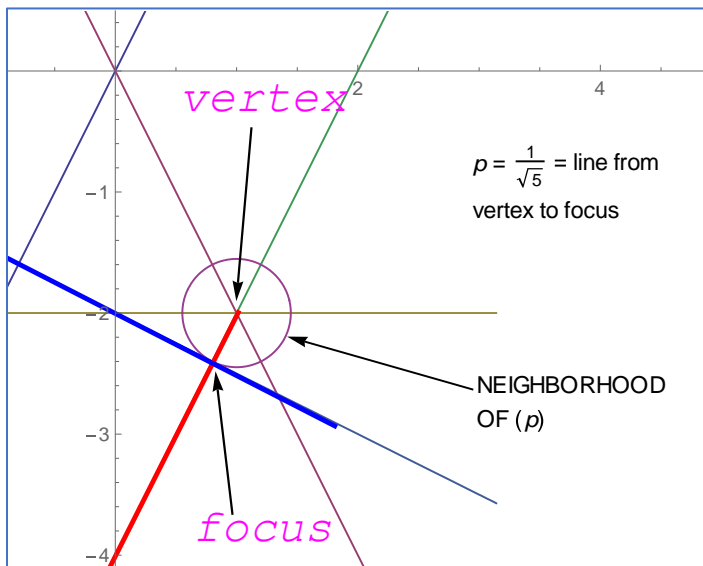
**SBG Theorem**: That line from the section focus to the <u>curve's</u> vertex will be the curve's initial focal radius ($p$) and have magnitude $\left( \dfrac{A}{m^2 \sqrt{\frac{1+m^2}{m^2}}} \right)$ where ($A = $ altitude) and ($m = $ cone slope). $\dfrac{2}{4\sqrt{\frac{5}{4}}} \xrightarrow{yields} \dfrac{1}{\sqrt{5}} = $ $\boxed{\{\frac{1}{\sqrt{5}} \text{Cos}[t] + 1, \frac{1}{\sqrt{5}} \text{Sin}[t] - 2\}}$

Neighborhood of ($p$) centered around vertex:

$$\text{ParametricPlot}[\{\{t, 2t\}, \{t, -2t\}, \{t, -2\}, \{t, -4 + 2t\}, \{t, \tfrac{1}{2}(-4 - t)\},$$

$$\{\tfrac{1}{\sqrt{5}} \text{Cos}[t] + 1, \tfrac{1}{\sqrt{5}} \text{Sin}[t] - 2\}\}, \{t, -\pi, \pi\}, \text{PlotRange} \rightarrow \{\{-2,6\}, \{\tfrac{-9}{2}, \tfrac{1}{2}\}\}]$$



With the section vertex location known, we can construct the "neighborhood of ($p$)". Where the neighborhood meets the principal and focal axis will lie the section latus rectum ($4p$) into the paper. The focus of the apollonian section is center to focal chord latus rectum. the value of ($p$), the sections initial focal radius, will run $2(p)$ +/- each way, into and out of the paper.

$p = \dfrac{1}{\sqrt{5}}$ = line from vertex to focus

*Figure 5*: Constructing the neighborhood of (p).

**STEP 6.** Determine location of the section focus to realize latus rectum *center.*

To find the ABSCISSA, set dependent axis parameters = to each other.

$$Solve[1/2\,(-4-t) == -4 + 2t, t] = \{t \xrightarrow{yields} 4/5\}$$

To find the ORDINATE set abscissa into either axis.

$$\frac{1}{2}(-4-t)\,such\,that\,(t)\,goes\,\frac{4}{5} = -(12/5)$$

LOCATION OF FOCUS $= (4/5, -12/5)$.

**STEP 7**. Next we determine the size of the diameter holding the curves <mark>focus</mark>. The ordinate of the section focus $(4/5, -12/5)$ is the new altitudinal level of the focal diameter. Use (A/m) to find the new radius and diameter of the section focus.

$\dfrac{\frac{12}{5}}{2} = \dfrac{6}{5}$. Radius = (6/5) and diameter = (12/5).

Altitude of focal diameter theorem: change of parabola section vertex altitude to focal altitude

$$= (A/(m^2 + 1)) = 2/5.$$

Change unit circle altitude placement from (2 units) to (10/5 units); we have:

 NEW ALTITUDE = (unit diameter altitude + change in altitude = new diameter.

$$\therefore \ 10/5 + 2/5 = (12/5).$$

NEW DIAMETER RADIUS = <mark>6/5 units</mark>.

Using proportional chord theorem, we find latus rectum chord, where (b) is a right triangle altitude chord (see step 8).

$$\text{Solve}\left[\left(\frac{10}{5} * \frac{2}{5}\right) == b^2, b\right] = \left(b \to \frac{2}{\sqrt{5}}\right)$$

this is half the latus rectum chord:  $\frac{2}{\sqrt{5}} * 2 = \frac{4}{\sqrt{5}} =$ latus rectum chord.

**STEP 8**. To confirm meter of the curves Latus Rectum, use the focus location constructed with the (Z) axis view into the paper. Let the section latus rectum be $(GH)$ and be parallel with x-axis. Let the (y-axis) be $\left(\frac{12}{5}\right)$ diameter normal with (x-axis). Parts; (y-axis) are $\left(abs\left(\frac{-6}{5}\right) + \frac{4}{5} + \frac{2}{5}\right) = \frac{12}{5}$. Using intersecting chord theorem from plane geometry:

$$(aF)*(Fb) = (GF^2) \xrightarrow{yields} \sqrt{\left(\frac{6}{5}+\frac{4}{5}\right)*\left(\frac{2}{5}\right)} = \frac{2}{\sqrt{5}} = \frac{LR}{2}.$$

$$\text{LATUS RECTUM} = 2*\frac{2}{\sqrt{5}} = \frac{4}{\sqrt{5}}$$

.

$$ParametricPlot[\{\{6/5\ Cos[t], 6/5\ Sin[t]\}, \{4/5, t\}\},$$
$$\{t, -2\pi, 2\pi\}, PlotRange \rightarrow \{\{-3/2, 3/2\}, \{-3/2, 3/2\}\}]$$
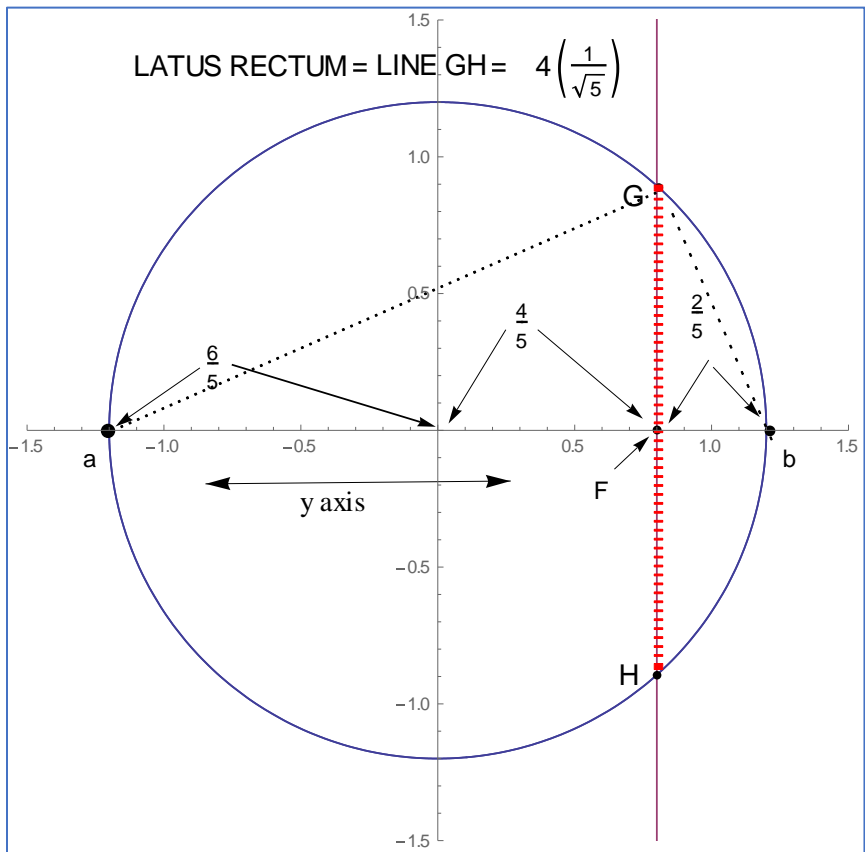


Chord (GH) is the focal chord Latus Rectum of the section parabola at conic altitude $\left(abs\left(\frac{-12}{5}\right)\right)$ and diameter (12/5).

*Figure 6:* The chord Latus Rectum of the section parabola is line GH and lies on the cone diameter holding the section focus.

**STEP 9**. Change curve character mapping from 2-space to 3- space.

# (x,y) Cartesian coordinates⟶(x,y,z) Cartesian System.

| object | ID | x | y |
|---|---|---|---|
| vertex | D | 1 | -2 |
| focus | $f$ | $\frac{4}{5}$ | $\frac{-12}{5}$ |
| vertex diameter center | C | 0 | -2 |
| focal diameter center | ☐ | 0 | $\frac{-12}{5}$ |
| latus rectum magnitude | $\frac{4}{\sqrt{5}}$ | ☐ | ☐ |

;

| object | ID | x | y | z | ☐ |
|---|---|---|---|---|---|
| vertex | D | 0 | 1 | -2 | ☐ |
| focus | $f$ | 0 | $\frac{4}{5}$ | $\frac{-12}{5}$ | ☐ |
| vertex diameter center | C | 0 | 0 | -2 | ☐ |
| focal diameter center | ☐ | 0 | 0 | $\frac{-12}{5}$ | ☐ |
| latus rectum end points | (+ / -) x | $\frac{\pm 2}{\sqrt{5}}$ | $\frac{4}{5}$ | $\frac{-12}{5}$ | ☐ |

**STEP 10.** Construct cone generators (π/2, π, 3π/2, and 2π).

$$ParametricPlot3D[\{\{0,t,2\ t\},\{0,-t,2\ t\},\{t,0,2\ t\},\{-t,0,2\ t\}\},\{t,-6,6\},$$
$$PlotRange-> \{\{-3,3\},\{-3,3\},\{-7,1/2\}\}, ViewPoint-> \{1,1,1\}]$$

We now use 3-D parametric graphing capability to construct a conic skeletal representation of a section parabola. I can only suggest repeat efforts to gain familiarity and imagined layout of parametric 3-D lines and curves.
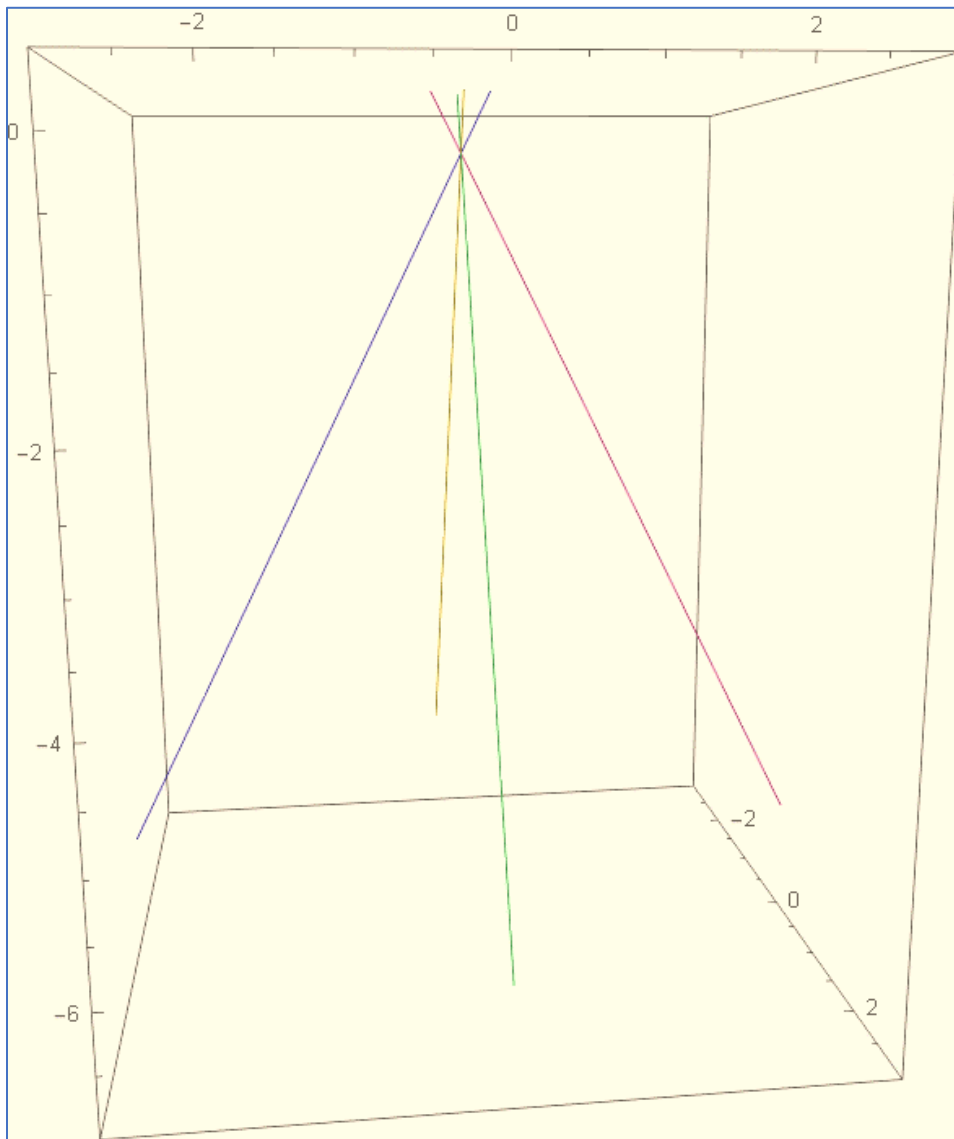


*Figure 7:* parametric construction of 4 generator; $\left(2\pi, \pi, \ \frac{\pi}{2}, and\ \frac{3\pi}{2}\right)$.

**STEP 11.** Construct diameters holding the focus and latus rectum.

$ParametricPlot3D[\{\{0, t, 2\ t\}, \{0, -t, 2\ t\}, \{t, 0, 2\ t\}, \{-t, 0, 2\ t\}, \{Cos[t], Sin[t], -2\},$
$\{6/5\ Cos[t], 6/5\ Sin[t], -12/5\}\}, \{t, -6, 6\},$
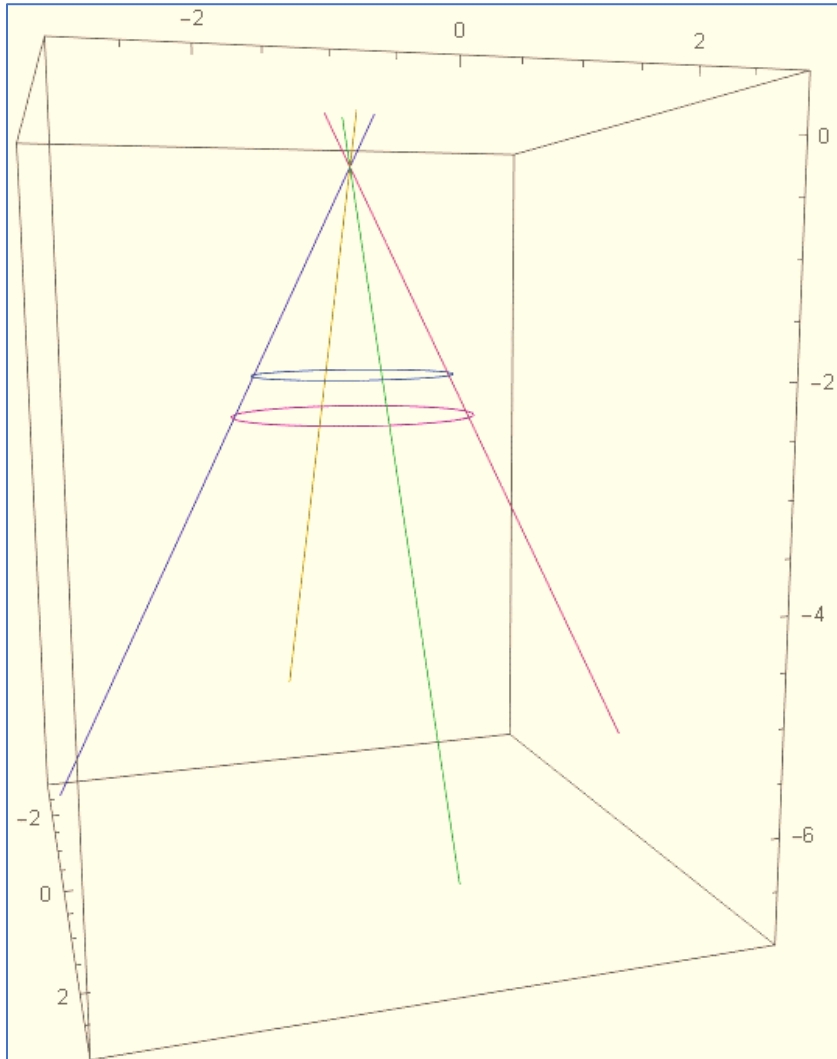$PlotRange-> \{\{-3,3\}, \{-3,3\}, \{-7,1/2\}\}, ViewPoint-> \{1,1,1\}]$



*Figure 8:* construct 2 cone diameters. One for section vertex and one for section focus.

unit circle diameter: $\{Cos[t], Sin[t], -2\}$.

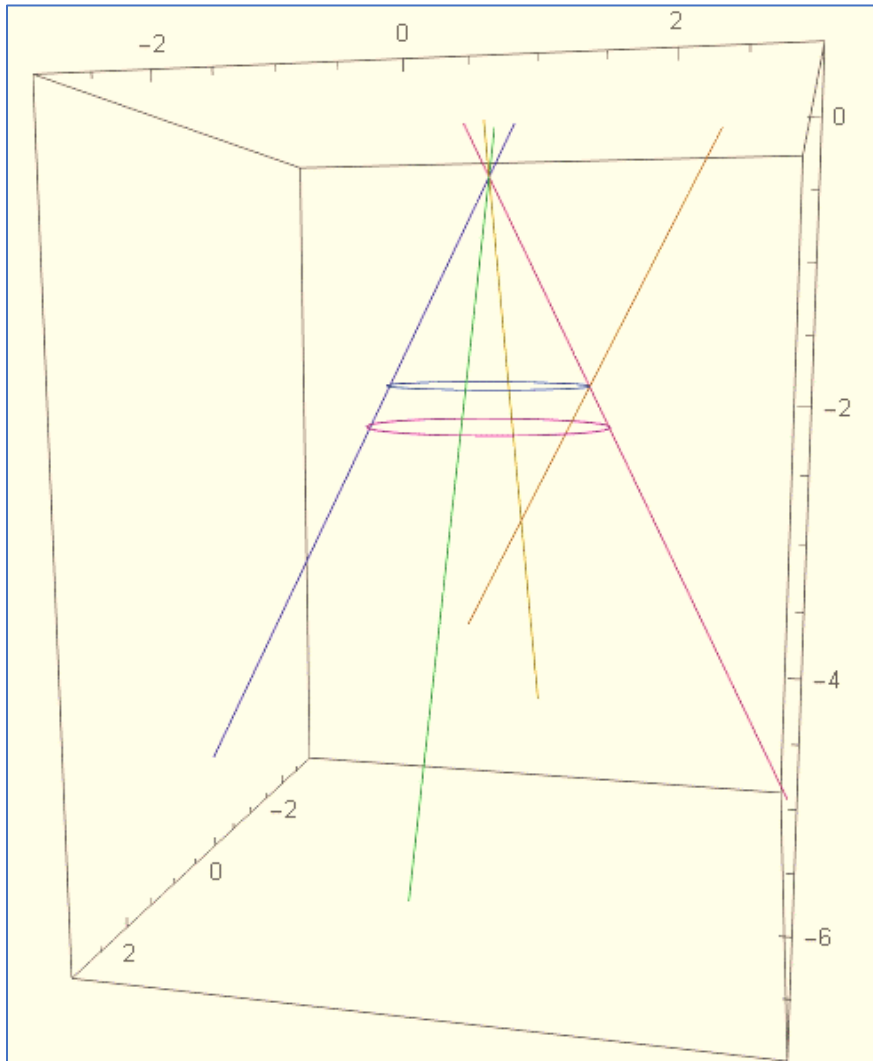Section focal diameter: $\{6/5\ Cos[t], 6/5\ Sin[t], -12/5\}$

**STEP 12.** Construct the Principal axis of the curve. Use the vertex and focus as linear locators (two points needed to make a line) and ask *Mathematica* to compute the direction number.

| vertex | □ | 0 | 1 | $-2$ | □ |
|--------|---|---|---|------|---|
| focus | □ | 0 | $\frac{4}{5}$ | $\frac{-12}{5}$ | □ |

$$\Rightarrow \left\{0-0,\ 1-\frac{4}{5},\ -2-\left(\frac{-12}{5}\right)\right\} \Rightarrow \left\{0,\ \frac{1}{5},\ \frac{2}{5}\right\} \Rightarrow \left\{0,\ 1+\frac{t}{5},\ -2+2\frac{t}{5}\right\}$$

$$\text{ParametricPlot3D}[\{\{0,t,2t\},\{0,-t,2t\},\{t,0,2t\},\{-t,0,2t\},\{\text{Cos}[t],\text{Sin}[t],-2\},$$
$$\{\frac{6}{5}\text{Cos}[t],\frac{6}{5}\text{Sin}[t],\frac{-12}{5}\},\{0,1+\frac{t}{5},-2+2\frac{t}{5}\}\},\{t,-6,6\},$$
$$\text{PlotRange} \to \{\{-3,3\},\{-3,3\},\{-7,\frac{1}{2}\}\}, \text{ViewPoint} \to \{1,1,1\}]$$



*Figure 9:* constructing the principal axis

Section principal axis:

$$\{0,1+\frac{t}{5},-2+2\frac{t}{5}\}.$$

Note how section principal axis touches unit circle diameter location of section vertex as was 2-space cone profile holding the cones principal axis.

STEP 13. Next we construct the focal axis. Use the vertex diameter center and focus as linear locators (two points needed to make a line) and ask Mathematica to compute the direction number.
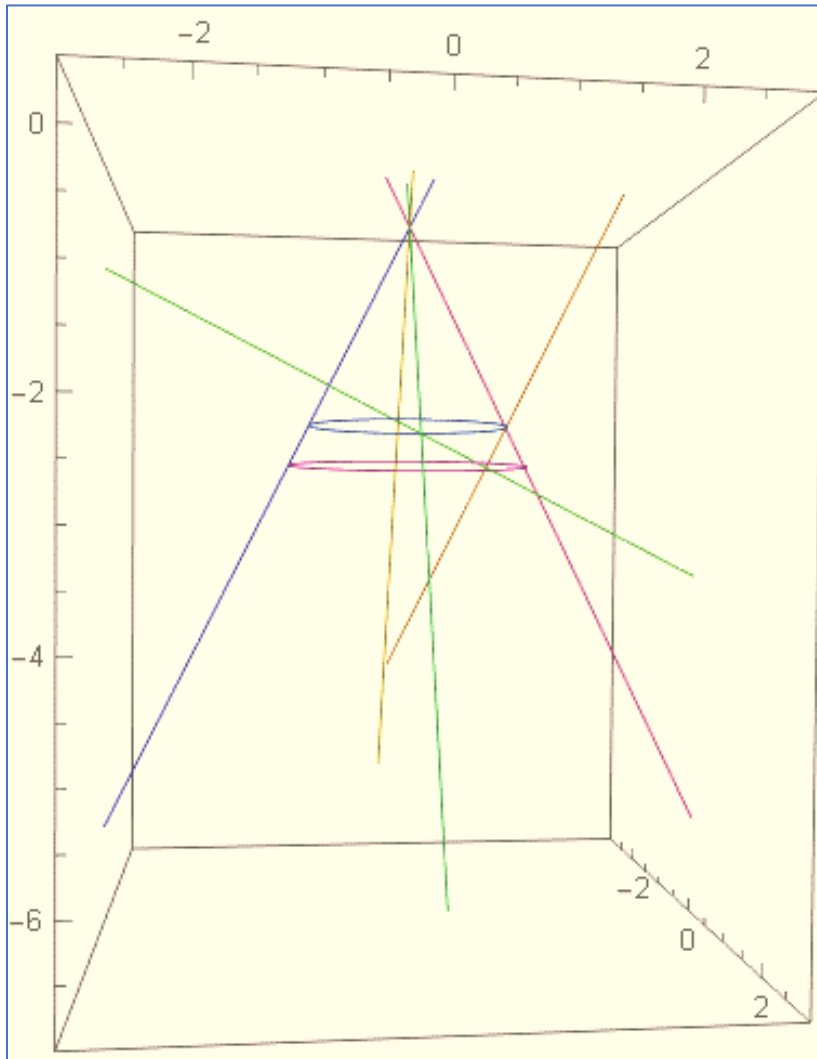
| focus | □ | 0 | $\frac{4}{5}$ | $\frac{-12}{5}$ | □ |
|---|---|---|---|---|---|
| vertex diameter center | □ | 0 | 0 | $-2$ | □ |

$(\text{focus} - \text{vertex}) \xrightarrow{\text{yields}} (\text{directionnumber})$

$(\text{focus} + \text{directionnumber} \xrightarrow{\text{yields}} (\text{parametric description of focal axis})$

$$\text{ParametricPlot3D}[\{\{0, t, 2t\}, \{0, -t, 2t\}, \{t, 0, 2t\}, \{-t, 0, 2t\}, \{\cos[t], \sin[t], -2\},$$
$$\{\frac{6}{5}\cos[t], \frac{6}{5}\sin[t], \frac{-12}{5}\}, \{0, 1+\frac{t}{5}, -2+2\frac{t}{5}\},$$
$$\{0, \frac{4}{5}+4\frac{t}{5}, \frac{-12}{5}+(\frac{-2t}{5})\}\}, \{t, -6, 6\}, \text{PlotRange} \rightarrow \{\{-3,3\}, \{-3,3\}, \{-7, \frac{1}{2}\}\},$$
$$\text{ViewPoint} \rightarrow \{1,1,1\}]$$



*Figure 10:* focal axis is normal with principal axis.

Section focal axis:

1.  $\{0-0, \frac{4}{5}-0, \frac{-12}{5}-(-2)\}$
2.  $\{0, \frac{4}{5}, -\frac{2}{5}\}$
3.  $\{0, \frac{4}{5}+4\frac{t}{5}, \frac{-12}{5}+(\frac{-2t}{5})\}$

$\{0, \frac{4}{5}+4\frac{t}{5}, \frac{-12}{5}+(\frac{-2t}{5})\}.$

STEP 14. Next we construct the latus rectum. Use the focus and either latus rectum end point as linear locators (two points needed to make a line) and ask *Mathematica* to compute the direction number.

| focus | □ | 0 | $\frac{4}{5}$ | $\frac{-12}{5}$ |
|---|---|---|---|---|

| latus rectum end points | □ | $\frac{\pm 2}{\sqrt{5}}$ | $\frac{4}{5}$ | $\frac{-12}{5}$ |
|---|---|---|---|---|

$$\{\frac{2}{\sqrt{5}}-0,\frac{4}{5}-\frac{4}{5},\frac{-12}{5}-(\frac{-12}{5})\} \xrightarrow{yields} \{\frac{2}{\sqrt{5}},0,0\} \xrightarrow{yields} \{\frac{2}{\sqrt{5}}+2\frac{t}{\sqrt{5}},\frac{4}{5},\frac{-12}{5}\}$$

ParametricPlot3D[$\{\{0,t,2t\},\{0,-t,2t\},\{t,0,2t\},\{-t,0,2t\},\{\text{Cos}[t],\text{Sin}[t],-2\},\{\frac{6}{5}\text{Cos}[t],\frac{6}{5}\text{Sin}[t],\frac{-12}{5}\},$

$\{0,1+\frac{t}{5},-2+2\frac{t}{5}\},\{0,\frac{4}{5}+4\frac{t}{5},\frac{-12}{5}+(\frac{-2t}{5})\},\{\frac{2}{\sqrt{5}}+2\frac{t}{\sqrt{5}},\frac{4}{5},\frac{-12}{5}\}\},$

$\{t,-6,6\},\text{PlotRange}\rightarrow\{\{-3,3\},\{-3,3\},\{-7,\frac{1}{2}\}\},\text{ViewPoint}\rightarrow\{1,1,1\}$]



I construct the latus rectum endpoint using the positive side of (x-axis).

$$\{\frac{2}{\sqrt{5}}+2\frac{t}{\sqrt{5}},\frac{4}{5},\frac{-12}{5}\}.$$

1.   $\{\frac{2}{\sqrt{5}}-0,\frac{4}{5}-\frac{4}{5},\frac{-12}{5}-(\frac{-12}{5})\}$
2.   $\{\frac{2}{\sqrt{5}},0,0\}$
3.   $\{\frac{2}{\sqrt{5}}+2\frac{t}{\sqrt{5}},\frac{4}{5},\frac{-12}{5}\}$
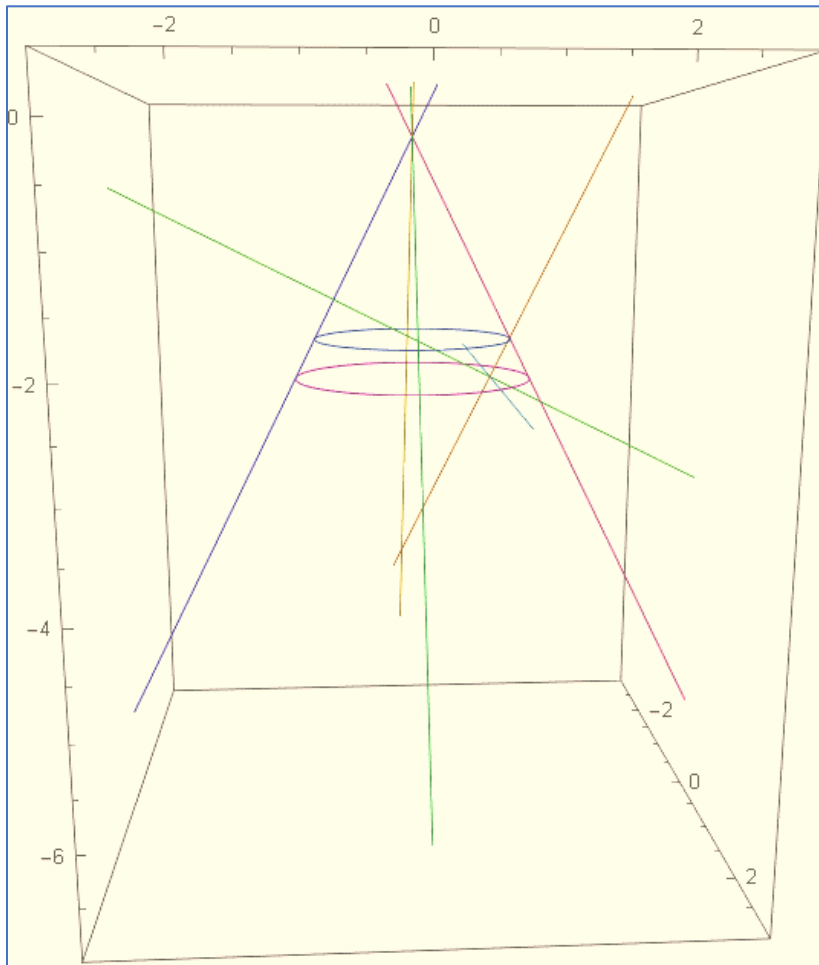
*Figure 11:* constructing the latus rectum of the section parabola.
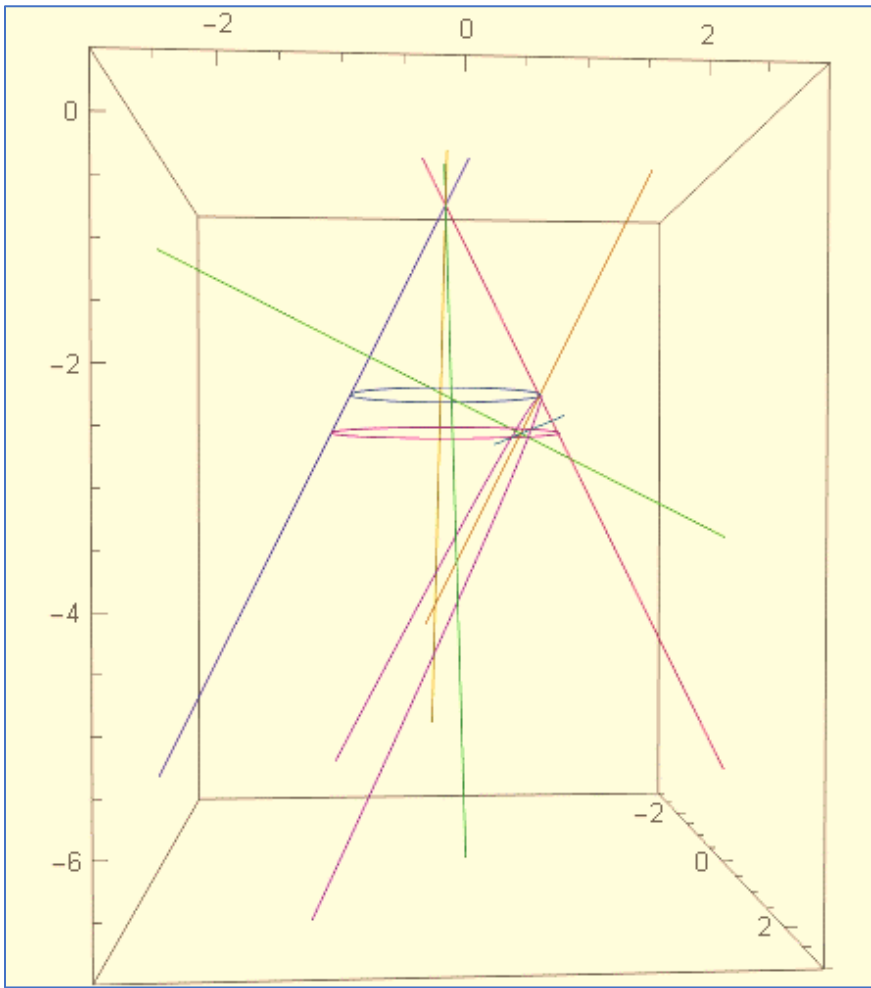
**STEP 15.** We now construct the Apollonian section using the following SBG identity.

$$\{t, \frac{t^2}{-4(\frac{A}{m})} + (\frac{A}{m}), \frac{m(t^2)}{-4(\frac{A}{m})} - A\} \Longrightarrow \{t, \frac{t^2}{-4} + 1, 2\frac{t^2}{-4} - 2\}$$

---

ParametricPlot3D[{{0, t, 2t}, {0, −t, 2t}, {t, 0,2t}, {−t, 0,2t}, {Cos[t], Sin[t], −2},

$\{\frac{6}{5}Cos[t], \frac{6}{5}Sin[t], \frac{-12}{5}\}, \{0,1 + \frac{t}{5}, -2 + 2\frac{t}{5}\}, \{0, \frac{4}{5} + 4\frac{t}{5}, \frac{-12}{5} + (\frac{-2t}{5})\},$

$\{\frac{2}{\sqrt{5}} + 2\frac{t}{\sqrt{5}}, \frac{4}{5}, \frac{-12}{5}\}, \{t, \frac{t^2}{-4} + 1, 2\frac{t^2}{-4} - 2\}\}, \{t, -6,6\},$

PlotRange → {{−3,3}, {−3,3}, {−7, $\frac{1}{2}$}}, ViewPoint → {1,1,1}]

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Parametric construction of section parabola (*m* = 2) cone.

$$\left(t, \frac{t^2}{-4\left(\frac{A}{m}\right)} + \left(\frac{A}{m}\right), \frac{m(t^2)}{-4\left(\frac{A}{m}\right)} - A\right)$$
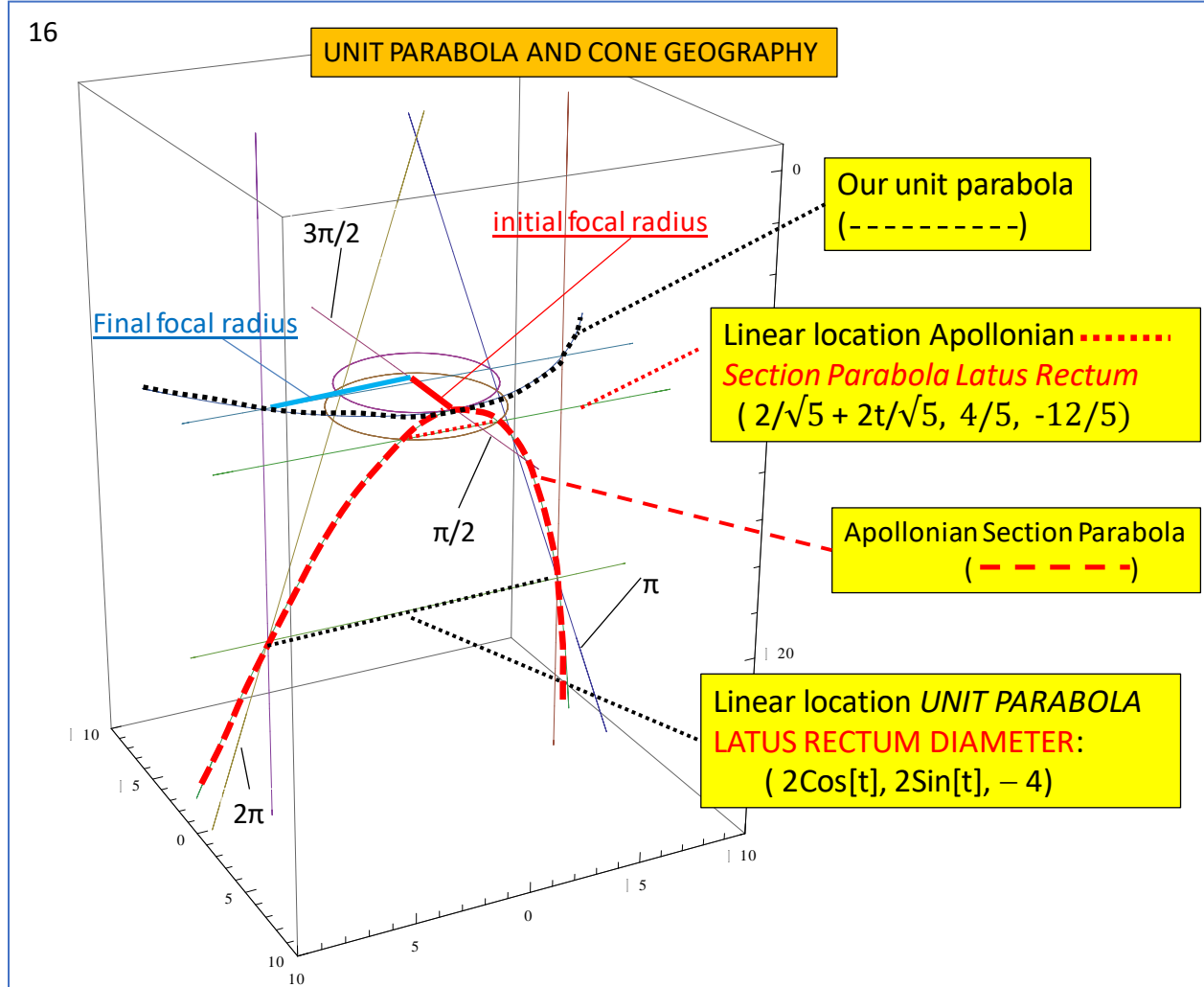
$$\{t, \frac{t^2}{-4} + 1, 2\frac{t^2}{-4} - 2\}$$

THE SECTION CURVE CAN BE COMPUTER GENERATED USING THE FOLLOWING **SBG** IDENTITY. PARAMETRIC DESCRIPTION OF THE APOLLONIAN SECTION PARABOLA IS HIGHLIGHTED.

Independent (t) will be x, and dependent (t) will be y composed as $\{\frac{t^2}{-4} + 1\}$, and the z axis composition will place section vertex at $\{\frac{2t^2}{-4} - 2\}$.

*Figure 12:* parametric computer-based construction of section parabola within a slope (*m* =2) right cone.

# Finding the Unit Parabola Duo-curve properties of the unit parabola



**16**

UNIT PARABOLA AND CONE GEOGRAPHY

3π/2

initial focal radius

Final focal radius

π/2

π

2π

Our unit parabola
( – – – – – – – – – –)

Linear location Apollonian ••••••••
*Section Parabola Latus Rectum*
( 2/√5 + 2t/√5,  4/5,  -12/5)

Apollonian Section Parabola
( – – – – – –)

Linear location *UNIT PARABOLA*
LATUS RECTUM DIAMETER:
( 2Cos[t], 2Sin[t], − 4)

16. Our unit parabola essentially has one and only one point on the cone surface, its vertex. With one end point of its focal radius on the cone central axis, a unit parabola active focal radius will provide two reads (*initial* and *final*) concerning changing cone surface curvature using the other end point trace of loci curvature. At the initial (red) unit parabola focal radius contact with the (pi/2) generator and section vertex on our unit circle, we find <u>cone surface curvature</u> is one and the same as the cone diameter holding the vertex: RoC = 1 unit. Read of the blue final focal radius at twice the altitude on the (2pi) generator intercept shows cone surface (RoC = 2 units), demonstrating changing cone surface curvature produced by iterate right cone diameters, from initial to final, are recorded on the unit parabola locus by our unit parabola focal radius.

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Sand Box Geometry LLC, a company dedicated to utility of Ancient Greek Geometry in pursuing exploration and discovery of Central Force Field Curves.

Using computer parametric geometry code to construct the focus of an Apollonian parabola section within a right cone.
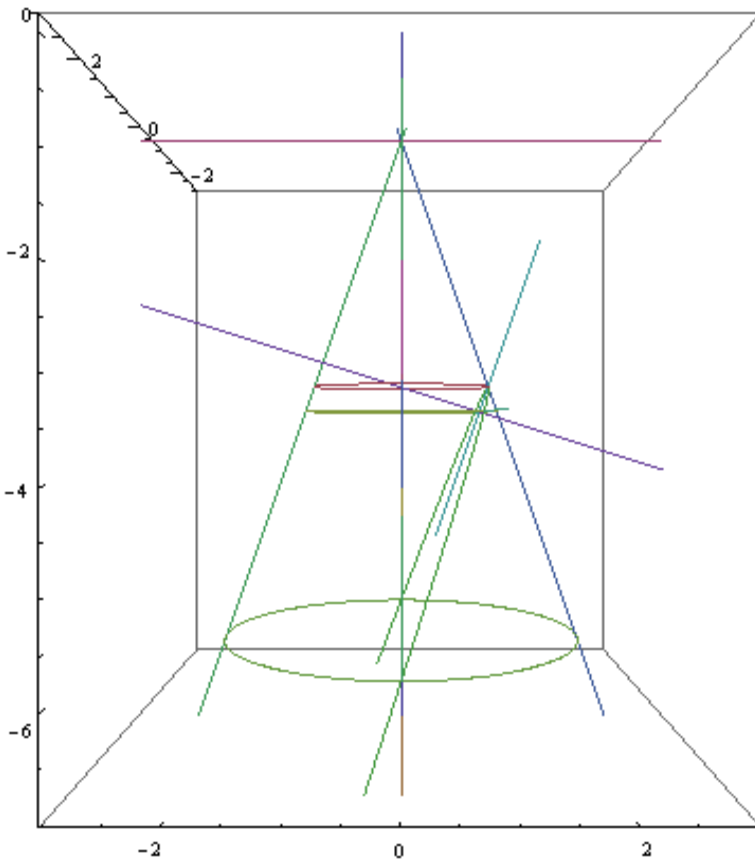


"It is remarkable that the directrix does not appear at all in Apollonius great treatise on conics. The focal properties of the central conics are given by Apollonius, but the foci are obtained in a different way, without any reference to the directrix; the focus of the parabola does not appear at all... Sir Thomas Heath: "A HISTORY OF GREEK MATHEMATICS" page 119, book II.

The computer is my sandbox, the unit circle my compass, and the focal radius of the unit parabola my straight edge.

ALΣXANDΣR; CEO SAND BOX GEOMETRY LLC

CAGE FREE THINKIN' FROM THE SAND BOX

The square space hypotenuse of Pythagoras is the secant connecting ($\pi/2$) spin radius (0, 1) with accretion point (2, 0). I will use the curved space hypotenuse, also connecting spin radius ($\pi/2$) with accretion point (2, 0), to analyze g-field mechanical energy curves.
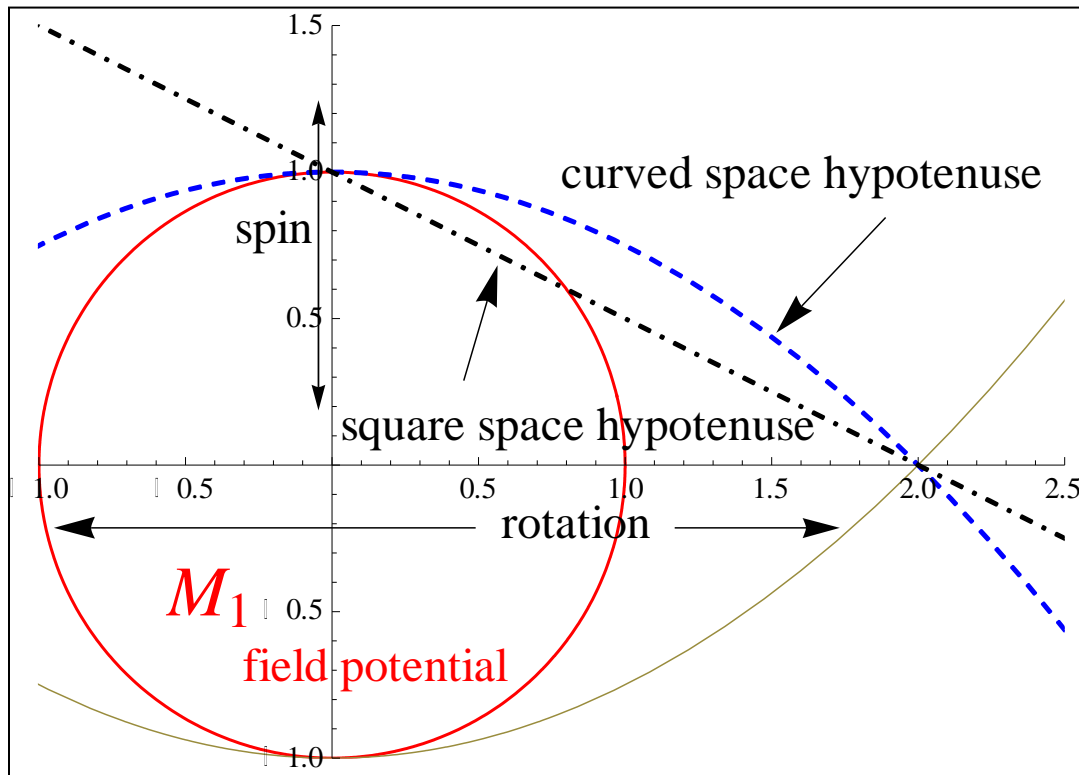


CSDA demonstration of a curved space hypotenuse and a square space

hypotenuse together.

We have two curved space hypotenuses because the gravity field is a symmetrical central force and will have an energy curve at the **N** pole and one at the **S** pole of spin; just as a bar magnet. When exploring changing acceleration energy curves of $M_2$ orbits, we will use the N curve as our planet group approaches high energy perihelion on the north time/energy curve.


ALΣXANDΣR; CEO SAND BOX GEOMETRY LLC